ELIT
Economic Laboratory Transition Research Podgorica

Montenegrin Journal of Economics

For citation:

Kurbatskii, A., Mironenkov, A. (2023), “Estimating the Quality of Life Using Weighted Principal Components Method”, *Montenegrin Journal of Economics*, Vol. 19, No. 1, pp. 7-17.

Estimating the Quality of Life Using Weighted Principal Components Method

ALEKSEI KURBATSKII[1] and ALEXEY MIRONENKOV[2] (***corresponding author***)

[1] Associate Professor, Head of Department of Econometrics and Mathematical Methods in Economics Lomonosov Moscow State University, Moscow School of Economics, Russia, e-mail: akurbatskiy@gmail.com
[2] Senior lecturer, Lomonosov Moscow State University, Moscow School of Economics, Russia. e-mail: alexey.mironenkov@gmail.com

ARTICLE INFO

Received March 11, 2022
Revised from April 09, 2022
Accepted May 10, 2022
Available online October 15, 2022

JEL classification:

C02, C10, C19, E24, E27.

DOI: 10.14254/1800-5845/2023.19-1.1

Keywords:

The weighted principal component method, the integrated indicator of Quality of Life, QoL and migration, countries’ ranking.

ABSTRACT

The paper proposes development of the approach to the construction of an integrated indicator of the population quality of life using the first principal component. We introduce the weighting coefficients into the principal component for the restoration quality of the initial features’ variation in the integrated indicator (II). That is, a number of variables is assigned more valuable in the first principal component and the preservation of variations of which in II is preferable. For the case of the II, it was shown that the target functional with weights can be reduced to the well-known problem of finding the first principal component without weights. Moreover, the paper proposes objective approach for finding these weighting coefficients based on the migration statistics. In accordance with this methodology original countries’ ranking based on the integrated indicator of Quality of Life (II QoL) was carried out.

INTRODUCTION

Correct and accurate measurement of the QoL is one of the most urgent tasks; a huge number of works is devoted to this topic, a very good overview of which is available in the work (Rakhmetova and Budeshov, 2020; Ulewicz and Blaskova, 2018). Usually, the process of constructing an integral indicator consists of two stages. At the first stage, an a priori set of initial variables is selected, then its unification and primary data processing is being done. The second stage is the construction of the "best representative" of this unified set. The choice of the initial set of indicators is determined by many factors, the requirements for it are described in detail, thoroughly and fully in the work (Aivazian, 2012). The choice of the "best representative" and correspondingly the final rating can be carried out in different ways. So, for example, in the work (Mironenkov, 2020), the final result is selected by sequentially applying the Pareto ratio; the work (Kobus, M, at. al, 2019; Aliyev, 2021; Aliyev et al., 2022) is based on stochastic dominance relations. Many papers use linear convolution with fixed weights as the main result (Blomquist et al, 1988; Moro et al, 2008; Belyaeva, 2009). In the works (Makarov et al., 2014; Leshchaikina, 2014; Aivazyan et al., 2016; Fantazzini et al., 2018; Shakleina and Midov, 2019; Afanasiev and Kudrov, 2019), the best representative of a unified set of variables is selected as the first principal component, modified principal component (Volkova, 2010) or generalized principal component (Volkova, 2019). Such a choice seems to be justified since the first principal component retains the maximum proportion of variation in the data set (Pearson, 1901). In other words, with the reverse restoration of the original data for the first principal component by the least square method, the sum of the squares of the deviations of the restored values from the original values is minimal. For more details look (Aivazyan and Mkhitaryan, 2001).

However, the use of the principal components in the quality of life analysis, with all the obvious advantages, has some problems (Jiang & Liu, 2021; Zheng & Jiang, 2021; Lapinskas et al., 2021). At the first stage, when compiling the initial (a priori) data set, the researcher has to take into account that each of the variables included in the set has the same value in the final rating. That is, despite the different values of the linear convolution weighting coefficients, the final integral indicator restores the original variables equally accurately. Or equally inaccurate. When a variable is included in an a priori data set, the researcher does not have the opportunity to set to each variable its own value (individual weight of reverse restoration). This in turn imposes restrictions on the choice of a set of variables. Thus, the introduction of weighting coefficients to improve the quality of restoration of some variables is seen as a natural direction for the development of the principal component method in the tasks of constructing an II QoL. At the same time, the quality of restoration of other variables may deteriorate.

1. WEIGHTED PRINCIPAL COMPONENT

At this stage we need to specify such a linear combination of the initial variables so that the OLS restoration of more important factors for this category is more accurate than the OLS restoration of less important factors. Thus, to build an integrated indicator based on the data $x_i^1, x_i^2, \dots, x_i^k$, where $i = 1, \dots, n$ the observation number, and k – is the number of variables, is suggested by predefined weights $w^1, w^2, \dots, w^k$, to build a linear combination $g = u^1x^1 + u^2x^2 + \dots + u^kx^k$, so that after the OLS-restoration of the original variables by the variable g and obtaining the restored variables $\hat{x}_i^1, \hat{x}_i^2, \dots, \hat{x}_i^k$ the following condition is fulfilled

$$w^1(\hat{x}^1 - x^1)^2 + w^2(\hat{x}^2 - x^2)^2 + \dots + w^k\left(\hat{x}^k - x^k\right)^2 \overset{u^1,\dots,u^k}{\rightarrow} \min, \qquad \text{(eq. 1)}$$

where x^1 is the first column vector of the source data, $\hat{x}^1$ is the column vector of the OLS restoration x^1 by column g.

So, let there be a matrix X of a posteriori set of unified variables of size $n \times k$, where n is the number of countries in the study (sample size), and k is the number of variables included in the posteriori set (number of features). It is assumed that $k < n$.

$X = \left(\begin{pmatrix} x_1^1 \\ \dots \\ x_n^1 \end{pmatrix} \ \dots \ \begin{pmatrix} x_1^k \\ \dots \\ x_n^k \end{pmatrix} \right)$ – a set of k variables, each of dimension n. For convenience, the lower index corresponds to the observation number, the upper index corresponds to the variable number.

Let G – $n \times m$ matrix of principal components, where m is the number of principal components used for analysis, i.e.

$G = \left(\begin{pmatrix} g_1^1 \\ \cdots \\ g_n^1 \end{pmatrix} \quad \dots \quad \begin{pmatrix} g_1^m \\ \cdots \\ g_n^m \end{pmatrix} \right)$ – matrix of m principal components.

Let $U = \left(\begin{pmatrix} u^{1,1} \\ \dots \\ u^{k,1} \end{pmatrix} \dots \begin{pmatrix} u^{1,m} \\ \dots \\ u^{k,m} \end{pmatrix} \right)$ – matrix of restoration coefficients.

Henceforth, constructing an integrated indicator, we assume $m = 1$, but the reasoning is valid for any $m \leq k$.

Due to the centering of a posteriori set, OLS-restoration of initial data using the principal components can be represented as $\hat{X} = G \cdot U^T$, where $\hat{X}$ is $n \times k$ matrix of recovered data, U^T is $m \times k$ matrix of data restoration coefficients (i.e., the matrix of coefficients for the principal components). Let there also be a diagonal $k \times k$ matrix $W = \begin{pmatrix} w^1 & & 0 \\ & \ddots & \\ 0 & & w^k \end{pmatrix}$, where $w^1, w^2, \dots, w^k$ – strictly positive values[1] of restoring the original variables.

The objective functional can be represented in matrix form as

$$\Delta_w(G, U) = \|\hat{X} - \mathrm{X}\|_w = \|\mathrm{G} \cdot U^T - \mathrm{X}\|_w = \mathrm{tr}\big((G \cdot U^T - X)^T \cdot W \cdot (G \cdot U^T - X)\big)$$

and the optimization problem (eq. 1) takes the form

$$\mathrm{tr}\big((G \cdot U^T - X)^T \cdot W \cdot (G \cdot U^T - X)\big) \to \min_{G,U} \qquad \text{(eq. 2)}$$

There is a problem statement (without solution) in equation B.16 in the work (Abdi, Williams, 2010); in the paper (Gabriel and Zamir, 1979) the same optimization problem is posed and an algorithm for its numerical solution is given; in the work (Burnaev and Chernova, 2008) there are a numerical solution and an iterative algorithm for solving a similar problem in determining the optimal wing profile. An iterative algorithm called "Heteroskedastic Matrix Factorization" is proposed in the study of the astrophysical spectrum (Tsalmantza and Hogg, 2012); and the paper (Delchambre, 2015) presents examples of the application of weighted principal components in astrophysics problems.

Note that the solution of the problem without weights

$$\mathrm{tr}\big((G \cdot U^T - X)^T \cdot (G \cdot U^T - X)\big) \to \min_{G,U} \qquad \text{(eq. 3)}$$

is well known, see, for example, equation B.15 in (Abdi, Williams, 2010), the matrices G and U can be found from the system

$$\begin{cases} G = \mathrm{XU} \cdot (U^T \cdot U)^{-1}, \\ U = (X)^T G (G^T G)^{-1}. \end{cases} \qquad \text{(eq. 4)}$$

And the components of the vectors U are obtained as normalized eigenvectors of the covariance matrix Σ (Jackson, 2005; Jolliffe, 2002; Dunteman, 1989).

You can easily check that the problem (eq. 2) can be reduced to the problem (eq. 3) by the following change of variables:

$$X_W = X \cdot V^T,$$

$$U_W = V \cdot U,$$

where $V^T V = \mathrm{W}$. Due to the positive definiteness of W such a change exists.

[1] If the restoration weight w^j is 0, that is, the restoration value of some variable is unimportant; it should be removed from the dataset.

Indeed, let us substitute in (eq. 3):

$$\mathrm{tr}\left(\left(G\cdot U_W^T - X_W\right)^T \cdot \left(G\cdot U_W^T - X_W\right)\right) =$$
$$= \mathrm{tr}\left(\left(G\cdot U_W^T - X_W\right) \cdot \left(G\cdot U_W^T - X_W\right)^T\right) =$$
$$= \mathrm{tr}\left(\left(G\cdot (V\cdot U)^T - X\cdot V^T\right) \cdot \left(G\cdot (V\cdot U)^T - X\cdot V^T\right)^T\right) =$$
$$= \mathrm{tr}\left(\left(G\cdot U^T\cdot V^T - X\cdot V^T\right) \cdot \left(G\cdot U^T\cdot V^T - X\cdot V^T\right)^T\right) =$$
$$= \mathrm{tr}\left(\left(G\cdot U^T - X\right) \cdot V^T\cdot V\cdot \left(G\cdot U^T - X\right)^T\right) =$$
$$= \mathrm{tr}\left(\left(G\cdot U^T - X\right) \cdot W\cdot \left(G\cdot U^T - X\right)^T\right).$$

In other words, the solution of the optimization problem with weights (eq. 2) with a data matrix X can be found as a solution to a standard problem without weights relative to the data matrix X_w. At the same time from (eq. 4) matrix G can be easily found.

2. METHODOLOGY

The above implies the possibility of generalization of considered method for a wide range of problems in which the integrated indicator is constructed as the first principal component and the weighted principal component can be constructed with pre-set weights. The question of obtaining weights w^j for the values of the variables remains open. The use of expert assessments does not correlate well with the "without a teacher" approach to obtain an integrated indicator. In the work (Fantazzini et al., 2021), it was noted that migration flows within the country are directed to the regions with a higher quality of life, usually to large cities. There are many sources that show the relationship between migration flows and human capital accumulation (Sardadvar, Vakulenko, 2021) or Happiness level (Porell, 1982; Michalos, 1996). Since, as shown in (Bartram, 2015), migration flows cannot directly serve as an indicator of the quality of life, migration statistics can be used as an objective teacher to determine the weighting coefficients for the values of variables in the principal component. In other words, it is supposed to pick up weight coefficients $w^1, w^2, \dots, w^k$ in such a way that the first principal component constructed with them most accurately corresponds to the real migration flows. The ratio of the number of migrants arriving to the number of those departing can be chosen as a measure of migration flows. It can be assumed that a higher value of the indicator will correspond to a higher level of quality of life. The correlation coefficient of the resulting integrated indicator with migration indicator is used as an objective function.

2.1 Data and Sources

In order to preserve continuity and the possibility of tracking dynamics, the a priori set of variables is identical to the one used in (Aivazian, 2012). There is also a very convincing list of conditions and data requirements for inclusion in the set. The list of variables is given in the Table 1.

Table 1. Selected predictors for II QoL.

№	Variable	Code WCY(2009)	Indicator
1	x^1	1.1.22	GDP per capita, PPP (USD).
2	x^2	3.1.04	Labor productivity, USD
3	x^3	1.1.23	Personal consumption expenditures per capita, USD
4	x^4	4.5.14	Literacy rate, %

5	x^5	2.5.07 / 2.5.06	20% coefficient of funds, times
6	x^6	1.5.01	Consumer Price Index, %
7	x^7	4.4.05	Life expectancy at birth, years
8	x^8	4.4.07	Infant mortality rate (per 1000 lives birth)
9	x^9	4.4.17	CO_2 emissions, metric tons
10	x^{10}	4.3.02	R&D expenditures, % of GDP

Unfortunately, the World Competitiveness Yearbook[2] (WCY 2009) database used in (Aivazian, 2012) is currently unavailable to us; the required indicators are obtained from the World Bank website[3] (WB 2019). At the time of the request (March 2020), variables x^2 and x^6 were available on the site for 2019 and variables x^1, x^3, x^4, x^8 for 2018, data on the other variables were presented for earlier periods. Herewith, countries containing a significant number of missing values were excluded from the sample. In case of a small number of omissions, the missing data was replaced by the available earlier ones. The latter took place in variables x^4, x^5, x^6, x^{10}. The World Bank also publishes statistics on migration (Bilateral Estimates of Migrant Stocks in 2017), at the moment of the access, data for 2017 are available. Since the initial data are presented in different units of measurement, to avoid the influence of dimensionality, we will carry out a standardization procedure $x^{new} = \frac{x-\bar{x}}{s(x)}$, where $\bar{x}$ is the mean value of the variable x, a $s(x)$ is the standard deviation.

2.2 Empirical findings

The resulting data matrix of dimension 103 by 10 is denoted by X. The correlation matrix of the original data set is shown in Table 2.

Table 2. The correlation matrix.

	x1	x2	x3	x4	x5	x6	x7	x8	x9	X10	MigrRate
x1	1,00										
x2	0,92	1,00									
x3	0,97	0,88	1,00								
x4	-0,66	-0,51	-0,66	1,00							
x5	-0,28	-0,30	-0,28	-0,12	1,00						
x6	-0,38	-0,39	-0,40	-0,28	0,09	1,00					
x7	0,64	0,71	0,68	-0,77	-0,35	-0,43	1,00				
x8	-0,51	-0,61	-0,54	-0,84	0,31	0,36	-0,93	1,00			
x9	-0,11	0,00	-0,10	0,31	0,08	0,02	0,10	-0,25	1,00		
X10	0,71	0,65	0,78	-0,39	-0,25	-0,34	0,59	-0,47	0,02	1,00	
Migr Rate	0,52	0,47	0,59	-0,17	0,00	-0,19	0,25	-0,17	0,04	0,49	1,00

[2] http://www.imd.org/research/books/world-competitiveness-yearbook-2019/

[3] https://data.worldbank.org/

We can see that migration flows are most strongly correlated with variables x^3 (Personal consumption) and x^1 (GDP per capita). First, let's build an integrated indicator as the first principal component without weights $g = u^1x^1 + u^2x^2 + \cdots + u^{10}x^{10}$, here $g = \min_{u^1,u^2,\ldots,u^k}(gU^T - X)^2$. The resulting coefficients u^j are shown in Table 3, and the resulting integrated indicator g and country ranks are given in the appendix.

Table 3. The coefficients of the first principal component.

U	u^1	u^2	u^3	u^4	u^5	u^6	u^7	u^8	u^9	u^{10}
PC1	0.379	0.386	0.387	0.313	0.163	0.218	0.387	0.355	0.031	0.331

The integrated indicator g, constructed as the first principal component, contains more than half (0.538) of the total variation of the original set. And the correlation coefficient of the first principal component with the migration indicator is 0.441.

To find the weighting coefficients of the quality of preserving the variation of the initial features, the following problem is solved:

$$\begin{cases} w^1, w^2, \ldots, w^k = argmax\big(cor(g; MigrationRate)\big), \\ g = u^1x^1 + u^2x^2 + \cdots + u^{10}x^{10}, \\ w^1(gu^1 - x^1)^2 + w^2(gu^2 - x^2)^2 + \ \ldots + w^k\big(gu^k - x^k\big)^2 \xrightarrow{u^1,\ldots,u^k} min. \end{cases}$$

The coefficients for variables in the principal component are given in Table 4.

Table 4. The coefficients of the first weighted principal component.

U	u^1	u^2	u^3	u^4	u^5	u^6	u^7	u^8	u^9	u^{10}
wPC1	0.447	0.404	0.464	0.222	0.131	0.188	0.318	0.255	0.039	0.390

Normalized optimal values of weights w^j are shown in Table 5.

Table 5. The coefficients of the first weighted principal component.

w	w^1	w^2	w^3	w^4	w^5	w^6	w^7	w^8	w^9	w^{10}
wPC1	0.000	0.000	0.975	0.000	0.000	0.000	0.000	0.000	0.000	0.224

At these values, the correlation coefficient reaches its maximum of 0.591. Note that the variable x^3, personal consumption expenditures, has the greatest value of preserving variation; the value of the variable x^{10} is a quarter of x^3 and all other variables are useless to maximize the relationship of the integrated indicator with migration indicators. Relationship between the correlation coefficient and the weights' ratio of the variables x^3 и x^{10} is shown in Figure 1. We see that the maximum correlation is achieved when $w^{10} = 0.23w^3$.

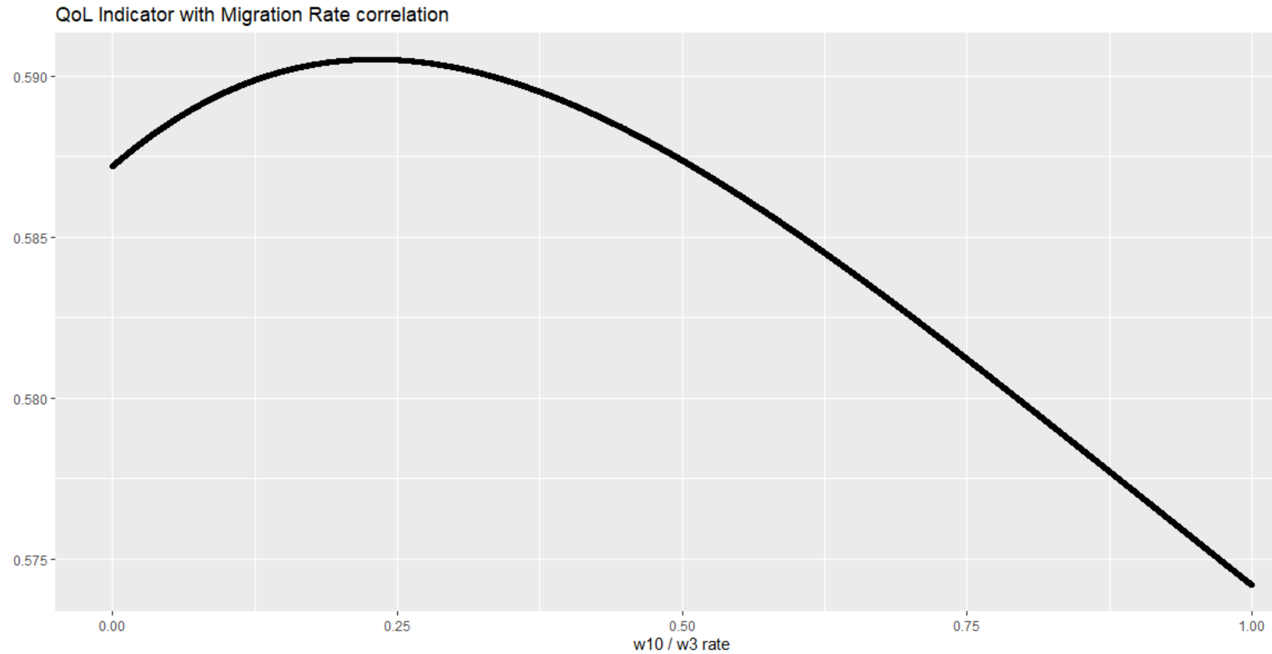

Figure 1. Dependence of the correlation coefficient on the ratio of weights.

The values of the integrated indicators obtained in this case, as well as the ranking, are given in the appendix.

DISCUSSION AND CONCLUSION

The paper has developed the principal component method in the applications for the QoL analysis, allowing the researcher to set the value with which the final integrated indicator should represent this variable when compiling an a priori data set. Comparing with the traditional principal component method, this approach has several advantages: the researcher has the opportunity to take into account the value of more important variables for his integrated indicator, while in the standard principal component method all the incoming factors are equally important. When constructing an integrated indicator, the researcher can consider (with an appropriately low weight) second-order factors that are incomparably less important than the main variables. Previously, the influence of such variables was ignored. This can make a significant contribution to the construction of II. It was possible to obtain a solution to the problem explicitly, without resorting to iterative evaluation procedures, thereby speeding up numerical calculations. As an application of the method, 103 countries were ranked in accordance with the generally accepted methodology.

As an interesting and relatively new approach we consider migration flows as an objective teacher for finding the corresponding weights. Of course, it is possible to introduce migration statistics in different ways and we use only one of them, anyway the final ranking results seem to be interesting and well consistent with the results of similar works known to the authors.

ACKNOWLEDGEMENTS

This research is supported by Russian Science Foundation: grant No. 20-68-47030_"Econometric and probabilistic methods for the analysis of financial markets with complex structure".

The authors express deep gratitude to their teacher Sergey Artemyevich Aivazian, who set the task.

APPENDIX.

Complete countries' ranking by the Migration Rate, Principal Component and weighted Principal Component.

Country	*Migr Rate*	*Rk*	*PC1*	*rk*	*wPC1*	*Rk*
Switzerland	3,950	12	4,755	2	5,970	1
Norway	4,111	11	4,534	3	5,612	2
Iceland	1,029	41	3,882	4	5,608	3
UnitedStates	15,652	1	3,756	7	5,492	4
Luxembourg	3,697	13	5,706	1	5,249	5
Denmark	2,150	24	3,836	5	4,593	6
Australia	11,856	2	3,312	12	4,114	7
Sweden	5,583	4	3,742	8	4,051	8
Finland	0,914	43	3,415	11	3,760	9
Austria	3,122	17	3,466	10	3,747	10
Israel	5,512	5	3,467	9	3,738	11
Germany	3,030	18	3,253	14	3,419	12
Belgium	3,335	15	3,310	13	3,414	13
Netherlands	2,004	26	3,170	15	3,293	14
Canada	6,328	3	2,773	19	3,180	15
UnitedKingdom	1,909	28	2,613	20	3,155	16
Japan	2,232	20	3,115	16	2,896	17
France	3,610	14	2,929	18	2,840	18
Ireland	1,085	39	3,835	6	2,652	19
Korea	0,510	55	3,020	17	2,223	20
Italy	1,825	29	2,374	21	1,962	21
Spain	4,306	9	2,039	22	1,411	22
Slovenia	2,221	21	2,006	23	1,031	23
Portugal	0,390	64	1,465	26	0,930	24
Greece	1,274	36	1,432	27	0,688	25
CzechRepublic	1,553	34	1,719	24	0,652	26
Estonia	0,668	48	1,393	28	0,554	27
Malta	0,427	59	1,699	25	0,520	28
Cyprus	1,158	38	1,288	29	0,370	29
Lithuania	0,342	68	0,834	34	0,260	30
SlovakRepublic	0,299	71	1,136	30	0,195	31
Uruguay	0,260	75	0,122	43	-0,026	32
Latvia	0,652	49	0,669	35	-0,035	33
Hungary	0,704	47	0,952	32	-0,064	34
Poland	0,155	84	0,989	31	-0,096	35
Croatia	0,602	53	0,879	33	-0,171	36
Chile	0,799	46	0,375	39	-0,263	37
Malaysia	1,717	31	0,524	36	-0,427	38
China	0,148	85	0,518	37	-0,513	39
Mauritius	0,340	69	-0,146	51	-0,520	40
Romania	0,107	86	0,308	40	-0,532	41
Brazil	0,431	58	-0,697	62	-0,537	42
CostaRica	3,171	16	0,034	45	-0,553	43
RussianFederation	1,063	40	0,080	44	-0,573	44
Turkey	1,676	33	-0,092	48	-0,744	45
Bulgaria	0,104	87	0,212	41	-0,753	46
Montenegro	0,345	67	0,383	38	-0,793	47
Serbia	0,822	45	-0,073	47	-0,815	48
Mexica	0,085	88	-0,378	53	-0,827	49
SouthAfrica	4,319	8	-2,149	85	-0,983	50
Botswana	2,007	25	-1,452	77	-1,027	51

Thailand	4,469	7	-0,010	46	-1,054	52
BosniaHerzegovina	0,023	102	0,177	42	-1,123	53
Jordan	4,278	10	-0,168	52	-1,132	54
Colombia	0,052	95	-0,724	63	-1,139	55
Kazakhstan	0,942	42	-0,139	50	-1,165	56
Ecuador	0,411	61	-0,580	58	-1,175	57
NorthMacedonia	0,232	77	-0,129	49	-1,191	58
Peru	0,072	92	-0,644	59	-1,213	59
Gabon	4,751	6	-1,097	72	-1,226	60
Tunisia	0,079	90	-0,660	60	-1,308	61
Paraguay	0,214	80	-1,114	73	-1,314	62
Iraq	0,192	81	-1,283	76	-1,330	63
Georgia	0,233	76	-0,518	55	-1,332	64
Morocco	0,036	99	-0,849	65	-1,351	65
Armenia	0,228	78	-0,411	54	-1,352	66
Guatemala	0,076	91	-1,503	78	-1,357	67
Eswatini	0,353	66	-2,624	87	-1,362	68
ElSalvador	0,027	101	-0,846	64	-1,366	69
Egypt	0,161	83	-1,812	81	-1,405	70
Moldova	0,395	62	-0,863	66	-1,410	71
Algeria	0,187	82	-0,529	56	-1,424	72
Ukraine	0,894	44	-1,026	69	-1,431	73
Kenya	2,501	19	-2,064	83	-1,437	74
SriLanka	0,030	100	-0,549	57	-1,478	75
Azerbaijan	0,424	60	-0,684	61	-1,481	76
Senegal	0,485	56	-2,178	86	-1,489	77
Vietnam	0,038	97	-0,907	67	-1,495	78
Indonesia	0,081	89	-1,082	71	-1,515	79
India	0,316	70	-1,824	82	-1,521	80
CaboVerde	0,065	94	-1,223	75	-1,536	81
Philippines	0,037	98	-1,153	74	-1,539	82
Mongolia	0,224	79	-1,005	68	-1,551	83
Ghana	0,609	52	-2,892	91	-1,562	84
BurkinaFaso	0,478	57	-3,087	93	-1,607	85
Honduras	0,049	96	-1,612	80	-1,624	86
Ethiopia	1,457	35	-3,211	96	-1,630	87
Nigeria	1,177	37	-4,506	102	-1,633	88
Pakistan	0,522	54	-2,851	90	-1,661	89
Nicaragua	0,067	93	-1,517	79	-1,662	90
Nepal	0,274	74	-2,067	84	-1,705	91
Lesotho	0,022	103	-4,085	101	-1,710	92
Mali	0,395	63	-3,695	100	-1,713	93
Coted'Ivoire	2,157	23	-3,599	98	-1,715	94
Sudan	0,367	65	-3,619	99	-1,716	95
Congo	1,683	32	-3,251	97	-1,719	96
KyrgyzRepublic	0,290	72	-1,034	70	-1,721	97
Chad	1,718	30	-4,615	103	-1,730	98
Togo	0,630	51	-2,984	92	-1,749	99
Gambia	1,925	27	-3,205	95	-1,779	100
Uganda	2,181	22	-2,675	88	-1,790	101
Burundi	0,642	50	-3,104	94	-1,840	102
Madagascar	0,284	73	-2,700	89	-1,857	103

REFERENCES

Abdi, H., Williams, L.J. (2010), "Principal component analysis", *Wiley interdisciplinary reviews: computational statistics*, Vol. 2, No. 4. 433–459.

Afanasiev, M., Kudrov, A. (2019), “Integrated Indicator of the Living Quality Conditions”, *Montenegrin Journal of Economics*, Vol. 14, No. 3, pp. 7–21.

Aivazian, S.A. (2012), *Analysis of the quality and lifestyle of the population: An econometric approach,* Nauka, Moscow (in Russian).

Aivazyan, S.A., Afanasiev, M.Y., Kudrov, A.V. (2016), “Clustering methodology of the Russian Federation regions with account of sectoral structure of the GRP”, *Applied Econometrics*, Vol. 41, pp. 24–46 (in Russian).

Aivazyan, S.A., Mkhitaryan, V.S. (2001), *Applied Statistics and Foundations of Econometrics, Probability theory and applied statistics, Vol. 1*, UNITY, Moscow (in Russian).

Aliyev, K., Gasimov, I., Eynalov, H. (2022), “Institutional trust and life satisfaction in selected Post-Soviet countries: The mediating role of 'perceived relative income”, *Economics and Sociology*, Vol. 15, No. 1, pp. 94-108. doi: 10.14254/2071-789X.2022/15-1/6

Aliyev, K. (2021), “Unemployment and (un)happiness: Life satisfaction approach to enhance policy efficiency for developing countries”, *Journal of International Studies*, Vol. 14, No. 4, pp. 220-235. doi: 10.14254/2071-8330.2021/14-4/15

Bartram, D. (2015), *Migration and quality of life in the global context. Global handbook of quality of life* (pp. 491–503), Springer.

Belyaeva, L.A. (2009), “The level and quality of life. Measurement and interpretation problems”, *Sociological research*, No. 1, pp. 33–42 (in Russian).

Belyaeva, L.A. (2018), “Quality of life in the subjective estimates of the population: Russia in the European context”, *RUDN journal of Sociology*, Vol. 18, No. 4, pp. 680–694 (in Russian).

Blomquist, G.C., Berger, M.C., Hoehn, J.P. (1988), “New estimates of quality of life in urban areas”, *The American Economic Review*, pp. 89–107.

Burnaev, E.V., Chernova, S.S. (2008), “On an iterative algorithm for calculating weighted principal components”, *Information Processes*, Vol. 8, No. 2, pp. 99–107.

Delchambre, L. (2015), “Weighted principal component analysis: a weighted covariance eigendecomposition approach”, *Monthly Notices of the Royal Astronomical Society*, Vol. 446, No. 4, 3545–3555.

Fantazzini, D., Pushchelenko, J., Mironenkov, A., Kurbatskiy, A. (2021), “Forecasting internal migration in Russia using google trends: Evidence from Moscow and Saint Petersburg”, *Forecasting*, Vol. 3, No. 4, pp. 774–804.

Fantazzini, D., Shakleina, M., Yuras, N. (2018), “Big Data for computing social well-being indices of the Russian population”, *Applied Econometrics*, Vol. 50, No. 2, pp. 43–66 (in Russian).

Gabriel, K.R., Zamir, S. (1979), “Lower rank approximation of matrices by least squares with any choice of weights”, *Technometrics*, Vol. 21, pp. 489–498.

Jackson, J.E. (2005), *A user's guide to principal components,* John Wiley & Sons.

Jiang, J., Liu, J. (2021), “A Study on Internal Quality Assurance Framework in Transnational Education”, *Transformations in Business & Economics*, Vol. 20, No 2B (53B), pp. 793-808.

Jolliffe, I.T. (2002), *Principal component analysis*, Springer New York.

Dunteman, G.H. (1989), *Principal components analysis,* No.69, Sage.

Kobus, M., Półchłopek, O., Yalonetzky, G. (2019), “Inequality and welfare in Quality of Life among OECD countries: Non-parametric treatment of ordinal data”, *Social Indicators Research*, Vol. 143, No. 1, pp. 201–232.

Lapinskas, A., Makhova, L., Zhidikov, V. (2021), “Responsible resource wealth management in ensuring inclusive growth”, *Polish Journal of Management Studies*, Vol. 23, No. 2, pp. 288-304.

Leshchaykina, M.V. (2014), “Econometric cross-country analysis of the living population social comfort”, *Applied econometrics*, No. 36, No. 4, pp. 102–117 (in Russian).

Makarov, V.L., Aivazyan, S.A., Afanasiev, M. Yu., Bakhtizin, A.R., Nanavyan, A.M. (2014), “The Estimation of the regions’ efficiency of the Russian Federation including the intellectual capital, the characteristics of readiness for innovation, level of well-being and quality of life”, *Economy of the region*, No. 4, pp. 9–30 (in Russian).

Maridal, J.H. (2017), "A worldwide measure of societal quality of life", *Social Indicators Research*, Vol. 134, No. 1, 1–38.
Michalos, A.C. (1996), "Migration and the quality of life: A review essay", *Social Indicators Research*, Vol. 39, pp. 121–166.
Mironenkov, A.A. (2020), "Hierarchical Pareto classification of the Russian regions by the population's quality of life indicators", *Economic and Social Changes: Facts, Trends, Forecast*, Vol. 13, No. 2, pp. 171–185.
Moro, M., Brereton, F., Ferreira, S., Clinch, J.P. (2008), "Ranking quality of life using subjective well-being data", *Ecological Economics*, Vol. 65, No. 3, pp. 448–460.
Pearson, K. (1901), "On lines and planes of closest fit to systems of points in space", *Philosophical Magazine 2*, pp. 559–572.
Porell, F.W. (1982), "Intermetropolitan migration and quality of life", *Journal of Regional Science*, Vol. 22, No. 2, pp. 137–158.
Rakhmetova, A., Budeshov, Y. (2020), "Quality of life as an indicator of public management performance in the Republic of Kazakhstan", *Economic Annals-XXI*, Vol. 183, No. 7/8, pp. 133–153.
Sardadvar, S., Vakulenko, E. (2021), "Does migration depress regional human capital accumulation in the EU's new member states? Theoretical and empirical evidence", *Review of Regional Research*, Vol. 41, pp. 95–122.
Shakleina, M.V., Midov, A.Z. (2019), "Strategic typologization of regions according to the level of financial independence", *Economic and social changes: facts, trends, forecast*, Vol. 12, No. 3, pp. 39–54.
Tsalmantza, P., Hogg, D.W. (2012), "A data-driven model for spectra: Finding double redshifts in the sloan digital sky survey", *The Astrophysical Journal*, Vol. 753, No. 2, 16 p.
Ulewicz, R., Blaskova, M. (2018), "Sustainable development and knowledge management from the stakeholders' point of view", *Polish Journal of Management Studies*, Vol. 18, No. 2, pp. 363-374.
Volkova, M.I. (2010), "Comparison of objectivistic and subjectivist approaches to measurement of synthetic latent categories of Quality of Life of the population: results of the empirical analysis of Russian data", *Applied Econometrics*, Vol. 19, No. 3, pp. 62–90 (in Russian).
Volkova, M.I. (2019), "Analysis of the factors of the quality of life of the population of Russia and Europe within the framework of the method of generalized principal components", *Economics and Mathematical Methods*, Vol. 55, No. 3, pp. 34–46 (in Russian).
Zheng, Y., Jiang, W. (2021), "Research on Translation Quality Self Evaluation in the Translation Process: Contents, Criteria and Support", *Transformations in Business & Economics*, Vol. 20, No 3 (54), pp. 317- 336.
Zhgun, T.V. (2017), "Building an integral measure of the quality of life of constituent entities of the Russian Federation using the principal component analysis", *Economic and Social Changes: Facts, Trends, Forecast*, Vol. 10, No. 2, pp. 214–235.