ELIT
Economic Laboratory Transition Research Podgorica

Montenegrin Journal of Economics

Citation:

Yerznkyan, B.H., Gataullin, T.M., Gataullin, S.T. (2023), "Mathematical Aspects of Synergy", *Montenegrin Journal of Economics*, Vol. 18, No. 3, pp. 197-207.

Mathematical Aspects of Synergy

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ARTICLE INFO

Received March 07, 2022
Revised from April 08, 2022
Accepted May 08, 2022
Available online July 15, 2022

JEL classification: A10, B41, C00, O40.

DOI: 10.14254/1800-5845/2022.18-3.16

Keywords:

synergy,
mathematical aspects,
consumption theory,
firm theory,
tax savings.

ABSTRACT

The paper is devoted to the concept of "synergy" and its semantic characteristics - an interdisciplinary direction of scientific research, synergistic effects, joint actions. The line between them is not always clear. And while the thought of synergy seems trivial enough, it has not yet penetrated the mass consciousness of economists. In the paper, attention is paid to such an understanding of synergy, which may be more in demand for economists and will help mathematically support some economic considerations. The paper considers four mathematical examples that can be used to express synergy: production synergy in the theory of the firm, synergy effect in the theory of consumption, tax savings and synergy in management.

INTRODUCTION: SYNERGY AND ITS SEMANTICS

We will talk about the fundamental property of the economy and, even more broadly, of the entire human civilization, especially under the new paradigm of the entrepreneurial economy (Sieja and Wach, 2019). This property is synergy. This word is used by scientific institutions, international scientific journals devote conferences to this concept, and this word is freely used in the general press. But it turns out that today there are two understandings of the term "synergy". Both of them appeal to the Greek word "*synergos*" – "jointly acting". It is difficult to decide which of them is earlier, more fundamental. There are three similar words that are often confused – these are synergetics, synergism and synergy. Synergetics is a science that studies the general laws of phenomena and processes in complex non-equilibrium systems. Synergism studies the self-organization of systems or the appearance of structures. It does not affect the synergistic effect that results from the appearance of synergy. Synergy is the equivalent of the concept of "*synergy*", it is used in works related to chemistry and biology. It is considered incorrect to use this term in the financial literature.

The paper will address the mathematical aspects of synergy and consider approaches to its understanding. Synergy is a complex concept that is currently receiving a lot of attention, and synergy can be quantified only in a few special cases. There are the following approaches to understanding synergy – the first approach was introduced by G. Haken, the second – by I. Ansoff. The paper will focus mainly on the second approach and touch a little on the first one.

We will conditionally consider the first understanding of synergy, introduced by the West German Professor G. Haken in 1969 (see: Haken, 1980). According to Haken, the meaning of the term "synergy" was that complex nonlinear systems are capable of self-organization and self-improvement. In this understanding of the term "synergy", Haken put two meanings: the first is the theory of the emergence of a new property in a whole consisting of actively interacting objects; the second is an approach that requires the cooperation of specialists from different industries for its development. The emergence of synergetics as an independent direction of scientific research can be dated back to 1969. It was then that the German physicist Hermann Haken began to use the term “synergetics" in his course on the theory of laser radiation, which he read at the University of Stuttgart. The new term was formed by him from the Greek expression Synergia, which means cooperation, concerted action, complicity.

What does the coordinated action effect have to do with the behavior of emitting atoms during the formation of a laser light beam? According to Haken, in a laser, a large number of atoms are immersed in an active medium, for example, in a crystal such as ruby. After being pumped from the outside, the atoms are excited and can emit separate arcs of light waves. Thus, each atom emits a signal, that is, it creates information carried by the light field. In the laser cavity, the emitted waves can collide with another excited atom, which will lead to an amplification of the wave emitted by it... Since individual atoms can emit light waves independently of each other, and since these waves can then be amplified by other excited atoms, a superposition of uncorrelated, though amplified, wave trains occurs, and we observe a completely irregular pattern. But when the signal amplitude becomes large enough, a whole new process begins. The atoms begin to oscillate coherently, and the field itself becomes coherent, that is, it no longer consists of separate uncorrelated wave trains, but turns into one almost infinitely long sinusoid.

Here is a typical example of self-organization: the temporal structure of a coherent wave arises without external interference. Chaos is replaced by order. A detailed mathematical theory shows that the resulting coherent light wave serves as a kind of order parameter, forcing the atoms to oscillate coherently, or, in other words, subjugates the atoms”. In the above passage, we will first highlight one concept — self-organization. It is the key to understanding the essence of synergy. Synergetics is defined as the science of self-organization or, more broadly, of the spontaneous emergence and self-maintenance of ordered temporal and spatial structures in open nonlinear systems of various natures.

In describing the process of coherent light wave formation, Haken uses a number of other fundamental concepts. Energy pumping means that the system in question is open, that is, it has an intense inflow of energy from outside, as well as outflows of energy. The resulting temporal or spatial structure is formed in the active environment and represents the identification of one of its potentially inherent discrete states. The system reacts non-linearly, that is, the transition from the disorganized behavior of atoms to the fusion of their radiation into a coherent light wave does not occur smoothly, in linear proportion to the increase in energy, but abruptly — at the moment when the energy inflow exceeds a certain barrier. The disjointed and disordered behavior of individual atoms corresponds to the chaotic state of the system, macroscopic chaos, from which order is born through a phase transition. For any system, you can define order parameters that allow you to describe its complex behavior in a fairly simple way, as well as select certain control parameters, when changing which significantly changes the macroscopic behavior of the system. The order parameters subordinate the behavior of individual elements of the system — which is the principle of subordination introduced by Haken.

Over the course of several decades, Haken's followers have written hundreds of articles and held numerous scientific meetings. This direction is associated with the names of I. Prigogine (1984) (Nobel laureate), G. Nikolis (1977, 1989), Soviet and Russian scholars: N.N. Moiseev, V. S. Stepin, S.P. Kapitsa, S.P. Kurdyumov, E.N. Knyazeva, D.S. Chernavsky and others. Synergy is a vivid example of international cooperation between scientists from different countries. In many countries around the world, there are research centers that explore various aspects of this understanding of synergy. In Germany, there is a Center for Synergetics at the Institute of Theoretical Physics, the Center is headed by Haken. The Prigogine School of Science operates in Brussels, in Paris, the Edgar Moreno Center. In our time, in this direction, synergetics is defined as an interdisciplinary direction of scientific research, in which the processes of transition from chaos to order and back (the processes of self-organization and self-organization) in open nonlinear environments of very different nature are studied (Knyazeva & Kurdyumov, 2005, 2007; Raisienė et al., 2019).

Synergetics includes the theory of dissipative structures (I. Prigogine, G. Haken), the theory of auto-oscillations and autowave processes (L.I. Mandelstamm, A.A. Andronov, R.V. Khokhlov, A.M. Zhabotinsky), the theory of "strange attractors" (Lorenz, 1992), B. Mandelbrot, the theory of "*catastrophes*" (R.V. Khokhlov; A.M. Zhabotinsky; Arnold, 1983; Gilmore, 1984), the theory of bifurcations of dynamical systems (Poincare, V.I. Arnold) and some other theories. We can also say briefly that synergetics is the theory of non-stationary rapidly developing systems. There are three key ideas of synergetics: self-organization, openness of systems, and non-linearity.

The openness of the system means the presence of sources or drains in the system, through which the exchange of matter or energy with the surrounding external environment is carried out. Non-linearity is the fundamental conceptual node of the new scientific paradigm underlying synergetics. Perhaps we can say that the new paradigm is the paradigm of non-linearity.

The described understanding of synergetics is now more fashionable than ever. Everything is included in synergetics – if something is not included in the classical established areas of research, then it is included in synergetics. The mathematical apparatus used in synergetics is also very complex – it includes everything that is possible: from unrecognized (and, perhaps, unscientific) concepts of time to the most complex chapters of the theory of differential equations.

One of the most well-known domestic developers of synergetics is the Nobel Prize winner in chemistry I. Prigogine (who left us in 2003 in Belgium). He gives the broadest interpretation of synergetics as an interdisciplinary science that studies the interaction of elements of different systems, leading to the emergence of spatial, temporal and spatiotemporal structures at microscopic scales. He considers synergetics as a theory of self-organization, based on such disciplines as the theory of catastrophes, system dynamics, the theory of dissipative structures, etc.

The main question of synergetics is: how does order arise from chaos, i.e. something stable, integral and harmonious; what gives mutual reinforcement of action; how does the whole produce a greater effect than the sum of the individual parts. According to I. Prigogine, the relationship between order and disorder characterizes scattered (dissipative) structures. Some of them develop in the direction of a "chaotic goal". A "*chaotic goal*" is neither a state of stability nor a mode of regular behavior. Apparently random, indeterminate walks occur. Therefore, in the process of evolution of two identical systems, the same conditions and causes do not lead to the same results. In the course of the development of economic and social systems, the same causes can cause similar effects only in very limited spaces and time intervals. All the rest of the time, historical development is carried out non-linearly.

Some researchers (and G. Haken himself among them) go even further and point out that for the first time the term "*synergetics*", or "*synergy*" (from the Greek *synergeia*), was introduced more than a hundred years ago by the English physiologist Sherington during the study of muscle systems and their control by the spinal cord. Since then, in biology, synergy refers to the effect of mutual complementarity of different bio-systems or individual species of flora and fauna. In a broad sense, this effect consists in the fact that the result of the activities of the elements combined in the system exceeds the sum of the results of the work of these elements acting separately. It is this understanding of synergy that the authors of the report will use in the future.

This second understanding of synergy is based on the fact that often the combination of parts into a single whole gives a greater effect than their mechanical sum. In this case, the synergistic effect is said to be positive.

This understanding of synergism is no less important than understanding the synergy of Haken.

The above definition gives the second understanding of synergy only a qualitative character. This definition is also based on the fact that often the whole has more reality than its parts. The quantitative measurement of synergy has been found so far only in some special cases. But even a qualitative understanding of synergy allows us to see the fundamental significance of this concept for the economy, moreover, for the entire civilization. In fact, anything designed, built, or functioning without a synergistic effect does not bear the stamp of perfection, on the contrary, it bears a tinge of inferiority. After all, in this case, the use, processing of resources, and in particular, human labor, occurs with a partial waste of their potentials, and it should be with their multiplication (Kosowski, 2020). It should be noted that human civilization has, in fact, one single resource-living human labor and it must be spent very carefully, achieving the necessary synergistic effect at each stage (Androniceanu at al., 2021; Wielechowski et al., 2021; Bareas et al., 2019).

The very idea of the need for a synergistic effect seems trivial, but in fact it has not yet penetrated the mass consciousness of economists and other specialists, although this understanding of synergy slips through in many publications, even in the public press.

In contrast to the first understanding of the term "synergy" of such a well-recognized founder as G. Haken, the second understanding of the term "synergy", perhaps, does not have. This understanding was born even earlier than the first one – in the 50s of the 20th century in the works of *inter alia* H.I. Ansoff (1989), K.I. Shilin (2003) on strategic planning and strategic management. Among the most famous foreign developers of synergy, it is possible to single out Ansoff, who proposed a fundamental concept that has practical significance. Having defined the economic basis of synergy as the possibility that the result of joint actions of several business units will exceed the final indicator of their independent activity, Ansoff considers both tangible and intangible assets as determining factors in close connection with the capabilities of companies. He identifies two forms of synergy: initial (savings at the stage of formation of the enterprise) and operational (savings at the stage of current activity), the measurement of which is associated with a certain combination of three variables: a higher amount of cash revenue from sales; lower operating costs and investment costs. We will give a very brief description of the second understanding of synergy under consideration.

It is also associated with such names as M. Porter (1985), H. Itami, (Campbell and Luchs, 1998) and others. M. Porter, one of the most authoritative scientists in the field of strategy, argued that managing the relationships between business areas (divisions, business units) is the essence of corporate-level strategy. According to the scientist, in the absence of rational management of relationships, a diversified company is "nothing more than a mutual fund". M. Porter examines the process of creating and strengthening a company's profitable positions in one or more industries based on the analysis of their value chain. The advantage of its concept is that it explains how material relationships or joint activities can achieve a competitive advantage, as well as helps to identify the synergistic potential and assess existing opportunities.

Japanese researcher H. Itami considers synergy as a process of increasing the efficiency of resource use. It defines two types of resources: physical (tangible) assets (such as production capacity) and invisible (intangible) assets. The latter are understood as intangible resources, which can include a brand name, consumer knowledge, technology, a strong, highly engaged employee, and a corporate culture. According to H. Itami, such invisible aspects, due to their uniqueness, are the best long-term source of the company's competitive advantages. They can not be purchased for money, they can be used and developed in different divisions of the company, as well as combined or used in new directions, ensuring the growth of the company.

The firm should strive to improve the results of using all available resources (Zheng et al., 2020; He et al., 2020; Iwashita, 2021). Ways to achieve this goal are to increase the utilization rate of all physical resources (expanding the product range without increasing production capacity) or to enter a new market in conditions of overproduction at the current one. By increasing the efficiency of the use of physical resources, the organization reduces costs, which is H. Itami defines it as a "complementary effect".

In theoretical terms, approach H. Itami's approach to synergy is very similar to the company's resource concept. Proponents of this concept argue that the company should not be considered as a discrete business, but as a set of tangible and intangible assets, or resources. The works of resource approach theorists emphasize the influence of internal and external sources (market position or industry attractiveness) of competitive advantages (Butkeviciene and Sekliuckiene, 2022). From this point of view, synergy as an effect that occurs in the process of using valuable internal resources is expressed in various end products.

Such scientists-specialists in strategic management K. Clark and K. Brennan offer a broader interpretation of synergy, according to which the corporation is divided into four portfolios: commodity, resource, consumer and technology. Each of them is analyzed using special matrices in accordance with certain criteria. Comparing the results of the analysis of corporate portfolios allows you to determine the potential for synergy of resources, products, consumers and technologies. For example, if the value of a certain group of customers increases, the corporation should focus its efforts on providing it with resources and technologies as a priority. When evaluating the planned integration processes, the company, according to the authors, should identify at least three or four areas of potential synergy, which will provide it with opportunities to use current strengths and develop new ones.

Despite the obvious advantage of synergy, its achievement, as evidenced by the experience of numerous foreign companies (see in fact the review: Campbell and Luchs, 1998), may be limited by the following circumstances:

- the possibility of obtaining a negative result if the combination (cooperation) leads to a non-optimal variant of the formed set of elements, which reduces the stability of the interaction and the equilibrium order;
- the complexity of the task of forming the structures of mechanisms and procedures necessary to create synergistic effects that require appropriate conceptual and methodological justification;
- the need to choose strict criteria for evaluating the effects of the development of quantitative methods for measuring them;
- the need for comprehensive consideration of the entire spectrum of emerging types and forms of synergistic effects, requiring their scientific classification and selection of relevant effects for the field of activity under study, etc.

Any economically organized system is based on certain elements that make up it. Each element has its own potential, its own functions and capabilities. The task of management is to combine the efforts of individual elements into a single mechanism and systemically organize their interrelated actions in such a way that the resulting end result exceeds the effect of independent actions. The activity of individual elements in joint activities will give greater effectiveness, i.e. the qualitative measurement of the result obtained will be superior to the quantitative measurement when all the individual components are added together.

In the process of functioning of complex economic systems, two opposite approaches to responding to the complexity of emerging problems have been developed. The first is based on the principle of "necessary diversity", proposed by R. Ashby. According to this principle, in order to cope with the increasingly complex problems of their environment, the complexity and speed of decision-making must correspond to the complexity and speed of changes in the external environment, i.e., it is necessary to develop increasingly complex organizational structures and management systems. The second approach is related to the principle of "*bounded rationality*", proposed by G. Simon and very similar to the above-mentioned principle of Ashby, according to which both individual managers and entire organizations are unable to cope with problems whose complexity exceeds a certain level corresponding to the ability of managers to make rational decisions.

The contradiction between these approaches is resolved on the basis of: reducing the complexity of the situation by simplifying the strategic position (getting out of unstable areas of activity); setting priorities for emerging problems and ensuring the next solution to the most important of them; reorienting the system or modifying its purpose if the solution to the problem it faces is too expensive or almost impossible; building hierarchical organizational structures to reduce diversity and identify the interaction of external conditions, as well as an orderly representation of the elements of the problem – the actors (stakeholders), their goals, criteria, strategies, alternative outcomes; the choice or development of more effective adaptive management systems (Yerznkyan, 2019; Lu et al., 2021; Prasalska-Nikoniuk et al., 2021).

Problems in which the predominant characteristic is "speed/surprise" are associated with an increase in the dynamics and a decrease in the predictability of processes occurring in the external environment. Such problems appear and develop too quickly to be able to prepare for them in advance and make the necessary decisions in time for the next planning cycle. Their impact causes two types of losses:

- related to the decline in operating profit (impact of the problem);
- the cost of loss prevention (reaction cost). The task of management is to minimize the total losses.

At relatively moderate rates of occurrence of new problems, comparable to the planning period, when solving them, the following methods are used: ranking of emerging problems and differentiated response to them in order of their urgency and importance; formation on the basis of previous experience of a model of success of actions, which allows you to quickly respond to changes in the external environment and choose an effective line of behavior; changing the type of management response to dynamic problems; improving the system of studying and diagnosing the external environment in order to increase the reliability of the information received and the speed of its processing, which reduces the response time to emerging problems.

An important methodological approach to solving such problems is to increase the strategic flexibility of the corporation (external and internal), the end result of which is to increase the ability to make effective management decisions. Strategic flexibility in the external environment is to diversify the economic activities of the corporation, which makes it possible to: provide additional growth in production and profit; reduce the risk of negative impact of possible problems. The internal flexibility of the corporation is expressed in the organization of its funds and structure, which allows you to quickly and effectively switch to the production of new types of products, to develop new markets. The elements of internal flexibility are: the flexibility of the management system, which allows you to respond to changes and emerging problems in a timely manner; the psychological readiness of management for changes; the liquidity of resources, a wide range of personnel qualifications and optimal management, a modular production structure, the readjustment of equipment and equipment!

Problems in which the predominant characteristic is "uncertainty/reliability" are associated with the possible risk of economic losses in achieving the final result. The uncertainty of the final result is influenced by the increasing complexity of relationships with the external environment, the unpredictability of the behavior of market participants, and the interdependence and variability of emerging problems. As the analysis has shown, problems do not exist in isolation, and therefore they cannot be isolated from the whole, explained separately, and integrated into the overall solution.

One of the possible ways to reduce uncertainty in solving complex problems is their hierarchical structuring, i.e. decomposition into components (elements) characterized by different nature and dynamic stability, as well as the gradual establishment of the strength of their relationship (priorities) in the hierarchy. Depending on the increase in the degree of uncertainty of the final result and the impossibility of its prediction, the management tools become more complex, and the role of risk management mechanisms increases.

As it is well known, the creation and functioning of integrated corporate structures is primarily aimed at obtaining a synergistic effect that ensures an increase in economic efficiency.

Returning again to the relationship between the two understandings of synergy, we state: it is very surprising – these two areas of research, two fairly large groups of researchers do not notice each other!

Thus, the monograph (Gataullin and Malykhin, 2007) provides an extensive bibliography of works on the first understanding of synergy, and among the 473 sources, neither Ansoff, Porter, nor Itami are mentioned! And in the no less fundamental monograph (Campbell and Luchs, 1998), in the list of about 200 scientists - researchers of the second understanding of synergy, Haken is not mentioned!

In this paper, we will deal with the second understanding of "synergy". Actually, it is closer to the economy than the first one. Moreover, it is important to emphasize that synergistic effects arise and must be maintained not only at "high" levels, but also, so to speak, in everyday life, at any level of management: at the enterprise level, at the level of municipal, city administration, etc.

1. PRODUCTION SYNERGY IN THE THEORY OF THE FIRM

The activity of any company can be described by the production function f. This function is shown and establishes a relationship between the resources (costs) that enterprise X uses and the output of Y. The production function is an increasing concave function of the cost vector. In the differential positive form, this can be expressed in the first partial derivatives of the function f and the negative definiteness of the Hesse matrix, which is composed of the second derivatives of the function f.

Output Y is usually expressed in kind if the company produces one type of product, and in money – if there is a total value of all products produced. The second approach can be reduced to the first if the unit price v is entered and in the second case v_1.

Thus , $Y = vf(X)$ is the income of this company or the revenue for the products sold. In the sense of the production function $f(0) = 0$.

As an example, let the firm expend a vector of resources $X = (x_1,...,x_n)$ (this is a column vector). Let the firm be able to X. The value $v(\partial f / \partial x_i)$ where the partial derivative is taken at the point X is called the marginal income (in the state X). In terms of content, this means that if another resource unit is involved in the state X, it brings additional revenue in the number of units $v(\partial f / \partial x_i)$. It is important to note that in the state X is not only the involvement of another unit of resource in production brings additional income in quantity $v(\partial f / \partial x_i)$, this is true for any unit of this resource-in particular, because of the indistinguishability in this theory of various units of resource, including living labor. It follows that the resource that is consumed in quantity can generate revenue in size x_i, and the sum of these revenues $V_i = v(\partial f / \partial x_i) \cdot x_i$ across all resources gives the value $\sum_i v(\partial f / \partial x_i) \cdot x_i$ (vector view - $vf'(X) \cdot X$, where $f'(X) = (\partial f / \partial x_1,...,\partial f / \partial x_n)$ (this is a vector-string)). Let this value be denoted $\sum_i V_i(X)$ and called the sum of income.

Now we can assume that the production function f is a linearly homogeneous function, i.e., $f(\lambda x_1,...,\lambda x_n) = \lambda f(x_1,...,x_n)$ or $f(\lambda X) = \lambda f(X)$. Linear homogeneity has the following economic meaning: an increase in λ costs by a factor entails an increase in output by the same number of times. It is given that a linearly homogeneous function satisfies the Euler equation $f(X) = f'(X)X$. This means that for such a function there is $vf'(X) \cdot X = vf(X)$, that is, the amount of revenue is equal to the revenue of this company.

The violation of the linear homogeneity of the production function can be interpreted as follows: let, for example, $f(X) > f'(X)X$. For such a function, it follows that the revenue is greater than the amount of revenue. With this outcome of events, it is said that the synergy effect in the joint (that is, complex) processing of resources is positive. It can also be concluded that the synergy of such a production function is positive.

Let's define the word "synergy»: the value $f(X) - f^{'}(X)X$ is called the synergy of the function f (at the point X) and is denoted $Cin(f)$ (or more precisely $Cin(f)(X)$).

We give the following two theorems without proof.

The first theorem is: the synergy of a linearly homogeneous function is 0.

The second theorem is: let be f is differentiable and $f(0) = 0$, then concavity f implies non-negativity of the synergy at any point. In particular, this is the position for the production function.

If $f(X) < f^{'}(X)X$, then they say that in this case there is a crowding effect. He says that the total processing of resources usually leads to a deterioration in the processing conditions of individual resources, and the effect of total processing is less than the sum of the effects of processing individual resources. In this case, the synergy is also said to be negative. Both (i.e., the synergy effect and the crowding effect) can be observed in reality. Sometimes a real production function gives a crowding effect, for example, when such a function is a Cobb - Douglas function $Y = AK^{\alpha}L^{\beta}$ and $\alpha + \beta > 1$ - see below. (Note that under this assumption, i.e., the Cobb-Douglas function $\alpha + \beta > 1$ is not concave).

Further, for a more complete representation, we should find the synergy of the Cobb-Douglas production function $AK^{\alpha}L^{\beta}$. We have: $Cin = AK^{\alpha}L^{\beta} - (\alpha AK^{\alpha-1}L^{\beta}K + \beta AK^{\alpha}L^{\beta-1}L) = AK^{\alpha}L^{\beta}(1 - (\alpha + \beta))$ and we see that when the synergy $(\alpha + \beta) < 1$ is positive, and when $(\alpha + \beta) > 1$ the crowding effect may appear, when $(\alpha + \beta) = 1$ the synergy is 0.

2. THE SYNERGY EFFECT IN THE THEORY OF CONSUMPTION

This effect shows a second mathematical illustration of synergy. The utility function u is an increasing concave function of the vector of the quantity of goods consumed X . In differential form, this can usually be expressed in the positivity of the first partial derivatives of the function u and the negative definiteness of the Hesse matrix, which is composed of the second partial derivatives of the function u (Gossen's first law). Here it should be assumed that $u(0) = 0$.

Let them consume a vector of goods $X = (x_1, ..., x_n)$. The quantity $\partial u / \partial x_i$ where the partial derivative taken at a point X is called the marginal utility of the commodity (in the state X), then $(\partial u / \partial x_i) x_i$ is the utility of the total quantity consumed of the commodity x_i , and $u^{'}(X) \cdot X = \sum_i (\partial u / \partial x_i) \cdot x_i$ is the sum of the utilities of all the commodities. The synergy effect in this case is as follows: the utility of a set of goods is greater than or, in extreme cases, equal to the sum of their utilities, i.e. when $u(X) >= u^{'}(X)X$. Since the utility function is assumed to be concave and differentiable, according to the second theorem, the synergy of the utility function is non-negative.

Further, it can be argued that another reason has been found for the production function and the utility function to be concave (this does not apply to the law of decreasing returns to labor and decreasing marginal utility with increasing consumption). First, we should explain why the production function should be concave. Any human civilization must organize its production in such a way that, in any case, it receives no less from the resources that a civilization has, than it "contains", i.e., from the resources that it has. so that the synergistic effect is non-negative (otherwise, the civilization may disappear). It follows from point 1 that this follows from the concavity of the production function. Next, why should the utility function be concave? For the same reason – thanks to evolution, man has learned to combine the consumption of products so that as a result of this consumption (Draskovic, 2022; Delibasic, 2022; Yerznkyan and Draskovic, 2022;), at least no less than they contain "singly" (in each individual product), this gives a non-negative synergistic effect.

3. SAVING ON TAXES

Tax savings are the third mathematical illustration of synergy. Under a certain tax system, it is advantageous to combine business units into one. A tax N is called infra-additive (see, for example) if $N(a)+N(b)\geq N(a+b)$. For the sake of specificity, let's assume that we are talking about income tax. Then it follows that even in a completely formal association, the tax paid by the association on the combined profits does not exceed the amount of taxes paid on the profits by the members of this association, and may be less. This is exactly what shows the manifestation of the synergy effect when combining. Note that an infra-additive tax can be called, for example, a tax that is set by a regressive scale. It is clear that such a tax encourages the merger of business units.

On the other hand, a tax N is called ultra-additive (see, for example: Gataullin, 2010; Gataullin et al., 2020) if $N(a)+N(b)\leq N(a+b)$. Again, for the sake of concreteness, we will leave the income tax. Then, even in a completely formal association, the tax paid by the association on the combined profits is not less than the amount of taxes paid on the profits by the members of the association, and may be even more. Note that ultra-additive can be called, for example, a tax set by a progressive tax scale. It is absolutely clear that such a tax encourages the separation of business units.

Next, we will draw up the specified synergistic properties of taxes in the following sentence:

Proposal 1. With a progressive (even ultra-additive) income tax, it is better for the taxpayer to employ several of his relatives and divide his salary between them - the total income tax will decrease; with a regressive (even ultra-additive) income tax, it is better to do the opposite.

4. SYNERGY IN MANAGMENT

4.1 General provisions

Assume that the energy of the system depends on two parameters $E=(a,i)$ and let the second parameter i is discrete. To bring the system to the state with the parameter i value, you need to expend energy $u(i)$. If $(a,i)>u(i)$, then this can be interpreted as a synergistic management effect. It is natural to achieve management with the maximum synergistic effect. Similar considerations can be developed in the case of a non-continuous parameter.

Example 1. The cost of increasing the unit of labor is u, and the production function is the Cobb-Douglas function $AK^{\alpha}L^{\beta}$. Need to study the synergetic effects of managing the volume of labor resources.

Decision. We will assume that the production function gives a monetary estimate of the product produced. Therefore, if $AK^{\alpha}L^{\beta}>uL$, then the synergistic effect of the control is positive and equal $\Delta(L)=AK^{\alpha}L^{\beta}-uL$. This effect is maximal when the derivative is $\frac{d\Delta(L)}{dL}=0$. We have $\frac{d\Delta(L)}{dL}=AK^{\alpha}(\beta L^{\beta-1}-u)=0$. Hence, the optimal L is $L^{*}=(u/\beta)^{1/\beta-1}$. Meaningfully, this means: find the volume of labor resources at which labor resources and fixed assets are in a good relationship with each other.

Example 2. The cost of increasing the unit of funds is u and the production function is the Cobb-Douglas function $AK^{\alpha}L^{\beta}$. To study the synergistic effects of managing the volume of funds.

Decision. We assume that the production function gives a monetary estimate of the product produced. Therefore, if $AK^{\alpha}L^{\beta} > uK$, then the synergistic effect of the control is positive and equal $\Delta(K) = AK^{\alpha}L^{\beta} - uK$. This effect is maximal when the derivative $\frac{d\Delta(K)}{dK} = 0$. We have $\frac{d\Delta(K)}{dK} = (A\alpha K^{\alpha-1} - u)(L^{\beta}) = 0$. Hence, the optimal K is $K^{*} = (u/\alpha)^{1/\alpha-1}$. Meaningfully, this means: find the volume of funds at which labor and fixed assets are in a good relationship with each other.

The considerations given in these examples are valid if the increase in resources or funds occurs from scratch.

It should be noted that the research topics we have mentioned only confirm the idea that the concept of synergy is central to all economic theory and is subject to further more thorough research.

CONCLUSION

In order to approach the mathematical aspects of synergy, which was the subject of the paper, it was first necessary to deal with the very concept of *synergy*. The fact is that, appealing to the Greek word "synergos", various scientists put into it similar, but irreducible concepts. Of these concepts, in which some focus on synergetics as an interdisciplinary science, others on the ability of the system to achieve a synergistic effect, and others on the possibility of cooperation, i.e. the joint action of the actors or other elements involved in the process, in the paper attention was paid to the second of them.

At the same time, it should be remembered that it is often problematic in practice to draw a clear distinction between the various semantic characteristics of synergy. The choice of the semantics we are interested in is explained by the fact that it turned out to be the most suitable for illustrating some of the economic and at the same time mathematically described problems presented in the paper: production synergy in the theory of the firm; the synergy effect in the theory of consumption; savings on taxes; synergy in management.

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