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New Modifications of Express Certificates on Two Assets

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ABSTRACT

The aim of the paper is to focus on modifications of express certificates. The paper shows the creation techniques of a new express certificate using two-asset correlation options and barrier options, which play the central role in financial engineering. Different possibilities of investment are investigated for the issuers and potential investor's point of view. Methodology of the paper is based on European style two-asset correlation options whose payoff is based on two underlying assets with two strike prices and barrier options. Due to the lack of real-traded two asset correlation options and barrier options, own calculations of option premiums are processed in MS Excel and statistical program R. Also, the pricing of the new express certificate with different parameters is examines with the showing of cost and profits for issuer and investor. Theoretical value of the modifications of express certificate with different levels of its parameters on the stocks Sanofi S.A. and Nestle S.A is obtained and it is performed the analysis of the profitability for to the issuer and investor at the maturity date. There is showed which parameters the investor should pay attention when deciding to invest into the given express certificate. Specific characteristics of proposed express certificate and its modification are pointed out and compared to each other. Also, minimum profit for issuer and for the investor in an ideal situation is presented. Proposed certificates can be part of a personal investment portfolio.

INTRODUCTION

As a result of the diversification of financial services in recent years, financial institutions have been able to release products that respond quickly to trends in financial or commodity markets. For this matter have been issued structured products for various financial assets or commodities in order to match their risk profiles to investors, both in terms of expected returns and in terms of risk response. According to Knop (2002), "a structured product is a financial instrument and its

return depends on the composition of other, simpler products. It consists of a loan and one or more derivative products'. According to the specific strategy can be designed proper products which are able to meet the expectations in any market (positive, stagnant, negative) and any risk profile (conservative, balanced, aggressive). It is, therefore, an innovative and flexible investment instrument that is an attractive alternative to direct investment and meets the investor's risk profile. Structured products are financial investments in which the amount paid to investors is exposed to different fluctuations in the reference value or the performance of one or several assets that this investor indirectly buys (Baranga, 2017). For more detailed information on structured products, see Azarmi (2017), Bluemke (2009), Knop (2002), Rossetto and Bommel (2009) or Yen and Kin Keung (2015).

The investment certificate is one of the types of structured products that are most traded on the market. Investment certificates based on two or more underlying assets include basket certificates (strategic and thematic). There are different variants of certificates, such as. cheapest-to-deliver certificates (assets are selected based on similar historical price developments, but one is issued with a larger discount), rainbow certificates (the highest weight is assigned to the asset class with the best return on average), bonus (Younis and Rusnáková, 2014), discount, outperformance (Hernandez et al., 2013), twin-win certificates and their modifications (with cap or barriers). Also, new form of alternative financing is crowdfunding which is presented by Soltes and Stofa (2016).

Since classical financial instruments, as one of the components of the product, are more well-known and not so complex, more attention is paid to options, specifically some types of correlation and barrier options belonging to a subset of exotic options. Hull (2018) defines vanilla option as a financial contract that provide its holder (the buyer) the right, but not the obligation, to buy (call option) or sell (put option) a given underlying asset at a specified future price (the strike price or the exercise price) of the option at any time within a specified expiration period of option (American style) or at the time of expiration of option (European style). The option premium is paid to the option seller (the writer) for this right. Exotic options have some different characteristics compared to vanilla options, however the essential features are the same. More detailed descriptions of classic vanilla and exotic options exist in the literature (Haug, 2007; Nelken, 1996; Zhang, 1998).

In this paper, we propose the new express certificate using the two-asset correlation options and barrier options (the type of exotic options). The proposal includes an analysis of potential profits from both the investor’s and the issuer’s points of view. In most cases, the issuer of investment certificates is a bank, but it may also be a brokerage firm that meets the legal capital requirements of particular countries. To the best of our knowledge, no study has yet utilized combination of two-asset correlation options and barrier options to create the investment certificate. This work therefore contributes to the literature by filling this gap, including practical application on two underlying assets Sanofi S.A. a Nestlé S.A. First of all, a short overview of express certificates and two-asset correlation options and barrier options is presented. This is followed by a creation analysis of new express certificate using these options. The final section contains the application on shares of S. Sanofi and S. Nestle.

1. THEORETICAL BACKGROUNDS

In the following part of the work we are going to pay attention to express certificate. The options which are used in the construction of express certificates are cash-or-nothing call options and down and knock-out put options. Their fundamental features are the fact that the profit of the options depends only on two underlying assets value at the maturity date. This is followed by two-asset correlation option and barrier option introduction and their valuation analysis.

1.1 Express certificates

Express certificate has a more complex structure. The maturity is usually between 3 and 6 years and the return is between 5 and 8 percent per year. Price movements are monitored at specific intervals (usually annually). Each year, the current value of the underlying assets is compared with a reference value, which is either the initial value at issue of the certificate (S_0) or the value that decreases by an agreed percentage each year. If the current underlying asset price is at or above this comparison level, the investor receives the nominal value plus the bonus (pre-agreed percentage) and the certificate is redeemed prematurely. Conversely, if it is below this level, the certificate continues to the next period and the process is repeated until the last observation, but the bonus is increased. A barrier is determined if the reference value is not reached even in the last observation. Barrier is crucial if the underlying asset price on the maturity date is below the reference level and the following two situations may occur: the value of the assets is below the reference level and at the same time below the barrier - the investor fully participates in the real decline of assets; the value of the assets is below the reference value and above the barrier - the investor receives the nominal value of the certificate. They are suitable for investors who expect the assets to move sideways or slightly rise and the volatility is reduced. At the same time, investors assume that assets will not break the barrier during the lifespan of the product.

Harcarikova and Banociova (2015) dedicated to the analysis of the classical express certificate for one underlying asset (Daimler AG and Allianz SE). This product was also investigated by Hernández, Tobler and Brusa (2010). Following the mentioned studies we design modification of express certificates on two underlying assets.

2. METHODOLOGY

Since options create a significant part of these products because they ensure the specific profile of the certificate, their valuation is especially important. At the same time, they form a risky part of the product as they can expire and lose their value. We used cash-or-nothing call options and down and knock-out put options to create an express certificate. Options are the significant part of every investment certificate, therefore our analysis must be based on these instruments. Two-asset correlation options are multiple asset options. They are options whose payoff is based on two underlying assets not necessarily belonging to the same class with two strike prices. The two assets are associated with one another through their correlation coefficient. There is not a derivative market for two asset correlation option. Due to the lack of market two asset correlation option prices, the values of the option prices was calculated. The prices of the options are calculated on the basis of available data of the chosen underlying assets from the Onvista financial portal, by the Haug pricing model (2007) in MS Excel and the R-studio program. In the case of every multi-asset option, there are more variables that have effect on the value of such an option. These variables are:

- S_1 a S_2 – underlying assets prices,
- S_{10} a S_{20} – underlying assets prices at the time of issue,
- X_1 a X_2 – strike prices for underlying assets,
- $S_1(\tau)$ a $S_2(\tau)$ – underlying assets prices at the time of expiration,
- T – time to expiration in years,
- B – barrier level,
- σ_1 a σ_2 – annualized volatilities of underlying assets,
- r – risk-free rate,
- δ – dividend yields of underlying assets,
- b – cost-of-carry of underlying assets, in the case of shares paying dividends: $b=r-\delta$,

– ρ – correlation between log-returns of underlying assets.

We used the Cash-or-nothing call option to ensure that the issuer was able to pay the agreed return on early repayment of the certificate. This option pays a fixed cash value (the cash settlement) of K if the prices of both assets on the maturity date are higher than their strike prices. The issuer may purchase at the same time several options at different expiry dates on precise cash amounts K. The calculation for the price of one such option on two underlying assets is expressed as:

$$c_{cash-or-nothing} = Ke^{-rT}M \quad (1)$$

where

$$d_{i,j} = \frac{\ln\left(S_i / X_j\right) + \left(b_i - \sigma_i^2 / 2\right)T}{\sigma_i\sqrt{T}} \quad (2)$$

a $M\left(d_{1,1}, d_{2,2}; \rho\right)$ represents the cumulative function of a two-component normal distribution, „bivariate normal distribution". Haug (2007) specifies the standard relationship for the calculation as:

$$M\left(d_{1,1}, d_{2,2}; \rho\right) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{d_{1,1}} \int_{-\infty}^{d_{2,2}} \exp\left[-\frac{x^2 - 2\rho xy + y^2}{2\left(1-\rho^2\right)}\right] dx\, dy \quad (3)$$

The result is the probability that one random variable is less than $d_{1,1}$ and the other random variable is less than $d_{2,2}$ when the correlation between the two variables is ρ. We applied this evaluation model to two selected underlying assets and performed the calculations in MS Excel.

Express certificates follow the barrier, which is monitored during maturity or at maturity date, depending on whether it is American or European option. We use the European type, which means that the barrier will not be monitored until the certificate's maturity date. The investor is thus provided with a kind of security buffer, where the asset can also fall, but only to a certain level. There is a distinction between call and put barrier options; down and up according to the barrier condition below or above the strike price; and when the barrier is reached/not reached, the option is activated (knock-in) or deactivated (knock-out). In our case, this is a down and knock-out put option, which is executed if the barrier is not broken and the price of the underlying is lower than the strike price. This allows you to buy an asset on the market for a lower amount and sell it at a higher strike price. The pay-out function can also be expressed analytically:

$$P_{down-out\,put} = \begin{cases} 0 & if\ S_i(\tau) \le B \\ X_i - S_i(\tau) & if\ S_i(\tau) > B \wedge S_i(\tau) < X_i \\ 0 & if\ S_i(\tau) > B \wedge S_i(\tau) \ge X_i \end{cases} \quad (4)$$

We calculated the theoretical prices of barrier options in the R-studio program and the functions used from the installed packages are based on the Haug pricing model (2007).

The price of the down and knock-out put options is calculated according to the relationship:

$$p_{down-out\,put} = A - B + C - D + F, \quad (5)$$

where

$$A = \Phi S_0 e^{(b-r)T} N(\Phi x_1) - \Phi X e^{-rT} N(\Phi x_1 - \Phi\sigma\sqrt{T}), \quad (6)$$

$$B = \Phi S_0 e^{(b-r)T} N(\Phi x_2) - \Phi X e^{-rT} N(\Phi x_2 - \Phi\sigma\sqrt{T}), \quad (7)$$

$$C = \Phi S_0 e^{(b-r)T} (B/S)^{2(\mu+1)} N(\eta y_1) - \Phi X e^{-rT} (B/S)^{2\mu} N(\eta y_1 - \eta\sigma\sqrt{T}), \quad (8)$$

$$D = \Phi S_0 e^{(b-r)T} (B/S)^{2(\mu+1)} N(\eta y_2) - \Phi X e^{-rT} (B/S)^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T}), \quad (9)$$

$$E = K e^{-rT} \left(N(\eta x_2 - \eta\sigma\sqrt{T}) - (B/S)^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T}) \right), \quad (10)$$

$$F = K\left((B/S)^{\mu+\lambda} N(\eta z) + (B/S)^{\mu-\lambda} N(\eta z - 2\eta\lambda\sigma\sqrt{T}) \right), \quad (11)$$

and for this option is valid that $\eta = -1, \phi = -1$ and at the same time the following relationship are:

$$x_1 = \frac{\ln(S/X)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}\ , \quad (12)$$

$$x_2 = \frac{\ln(S/B)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}\ , \quad (13)$$

$$y_1 = \frac{\ln\left(B^2/(SX)\right)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}\ , \quad (14)$$

$$y_2 = \frac{\ln(B/S)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}\ , \quad (15)$$

$$z = \frac{\ln(B/S)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}\ , \quad (16)$$

$$\mu = \frac{r - q + \sigma^2/2}{\sigma^2}, \quad (17)$$

$$\lambda = \sqrt{\mu^2 + 2r/\sigma^2}\ . \quad (18)$$

Among the best known authors who deal with options and their valuation are Bouzzobaa and Osseiran (2010), Buchen (2012), Haug (2007), Hull (2018), Wystup (2017) or Zhang (1995; 1998).

2.1 Proposal of express certificates using correlation options and their application

In this section, we propose modification of express certificate on two underlying assets. The proposal includes an analysis of potential profits from both the investor and issuer points of view. Also different levels of its parameters are shown on which the investor should pay attention to when deciding to invest into the given certificate. In our case, the certificate consists of three main components: Zero-coupon bond, correlation options – Cash-or-nothing call options on two underlying assets and down and knock-out put barrier options on both assets individually. Also, Gicova (2019) deals with proposed express certificates in theoretical way.

2.2 Data

The product is designed and valued at the start date on 15.02.2019 using real market data. The barrier is set at 60% of the starting value, which allows the share price decline up to 40%, while the investor assumes that no major decrease will occur. It is important to note that the barrier is not observed continuously, but only on the final valuation date 15.02.2022. The product is issued on 18.02.2019 with a nominal value of EUR 1,000 with a 3-year expiry date and a maturity date of 18.02.2022. During the subscription period there is also an entry fee, which in our case is 1% of the invested nominal value of the certificate. Early redemption may occur during the life of the certificate when the investor obtains a predetermined return under certain conditions. Early repayment yields, depending on when the conditions are met, are as follows: 106% after the first year, 112% after the second year and 118% of the certificate's nominal value after the third year of the certificate. Observation days are also known (14.02.2020 (1st year), 15.02.2021 (2nd year) and 15.02.2022 (3rd year)), when the current assets prices are compared with their reference values (termination levels), which are the same for each observation year.

The underlying assets are shares of European companies Sanofi S.A. and Nestle S.A.. They are assets of the same class, but from different sectors. Both companies belong to the largest in the industry and operate internationally, even worldwide.

Table 1. Common data about new express certificate "Express SanNes"

Name	*„Express SanNes"*	
Nominal value:	*1000*	
Currency:	*EUR*	
Investment horizon:	*3 years*	
Initial valuation date:	*15.02.2019*	
Issue value date:	*18.02.2019*	
Annual valuation dates:	*14.02.2020*	
	15.02.2021	
	15.02.2022	
Final valuation date:	*15.02.2022*	
Maturity date:	*18.02.2022*	
Settlement method:	*financial*	
Starting value:	*closing price of the shares on 15.02.2019*	
Barrier level:	*observation only at the final valuation date*	
Underlying assets	*Sanofi SA*	*Nestlé SA*
Starting value:	$S_{10} = 75,25$	$S_{20} = 78,64$
Reference value:	$X_1 = 75,25$	$X_2 = 78,64$
Barrier level (60 %):	$B_1 = 45,15$	$B_2 = 47,18$
Annualized volatility:	$\sigma_1 = 0,2187$	$\sigma_2 = 0,1535$
Dividends:	$\delta_1 = 0,0401$	$\delta_2 = 0,0278$
Cost of carry:	$b_1 = -0,03191$	$b_2 = -0,01961$
Correlation:	$\rho = 0,3382$	
Risk free interest rate:	$r = 0,00819$	

Source: own design based on data from Onvista

The first underlying asset, referred to as "1", is the shares of company Sanofi S.A, which was created by the gradual merger of pharmaceutical companies at international level. The initial price of Sanofi S.A is fixed at EUR 75.25 on 15/02/2019 with a barrier of 60% of this value, which is EUR 45.15. The annualized volatility of daily returns is 21.87%. The second underlying is denoted by index "2". Nestlé S.A is currently the world's largest food and beverage company. Its annualized

volatility is 15.35%. At the valuation date of 15.02.2019 the initial reference price was EUR 78.64. The barrier, which allows 40% decline on the final valuation date, is EUR 47.18. The correlation of the daily yields of Sanofi and Nestlé over the ten-year period, as measured by the correlation coefficient, is 0.3382. This value indicates that the assets are slightly positively correlated. Companies are from another industry and are not property linked. Sanofi S.A. and Nestle S.A. pay dividends at the moment. An overview of the designed product is shown in the Table 1.

During the construction we do not assume any transaction costs connected with the purchase of individual components of the proposed certificate and that there is a possibility on the market to carry out such operations. The risk-free interest rate is 0.819% on 15.02.2019 and is considered constant over life of the certificate.

3. RESULTS

The first component is Zero coupon 3-year bond with a 5% discount rate, with a nominal value of EUR 1,000. This bond is considered as a risk-free investment. The issuer would have bought this bond for the actual value of EUR 887.70 (considered with a three-year horizon and a current tax rate of 19%).

$$D = \frac{1000}{\left(1 + 0.05 \cdot 0.81\right)^3} = 887.70 \qquad (19)$$

To ensure that the issuer is able to pay the investor the agreed yield in each possible year of early repayment, the issuer realizes the following operations on both the underlying assets:

- purchase cash-or-nothing call options for possible payouts K=60 on 1 year (T=1) at EUR 13.08,
- purchase cash-or-nothing call options for possible payouts K=120 on 2 years (T=2) at EUR 22.28,
- purchase cash-or-nothing call options for possible payouts K=180 on 3 years (T=3) at EUR 29.30.

Based on the nature of the certificate the issuer realizes following barrier option operations on individual underlying assets in order to secur a risk buffer in the form of a barrier level:

- purchase of the European down and knock-out put option with a strike price $X_1 = S_{10}$, a barrier level $B_1 = 0{,}60S_{10}$ and the expiration time T=3, which is identical with the expiration of the express certificate at EUR 4.80;
- purchase of the European down and knock-out put option with a strike price $X_2 = S_{20}$, a barrier level $B_2 = 0{,}60S_{20}$ and the expiration time T=3 at EUR 7.03.

To purchase the individual components, the issuer uses the gained capital from the investor in the form of the issue price of the product. The cost on buying options is in at the amount of EUR 76.49. However, the issuer's total costs must also include the current purchase price of the bond at EUR 887.70, which is a total of EUR 964.19. The issuer will still have EUR 35.81 from the nominal value of EUR 1,000, which may be increased if a fee has been set during the subscription period. In our case, 1% of the nominal value, which means EUR 10. In fact, the issuer's lowest yield is EUR 45.81, what is 4.58%. Figure 1 represents the scheme of the proposed 3-year "SanNes Express" certificate.

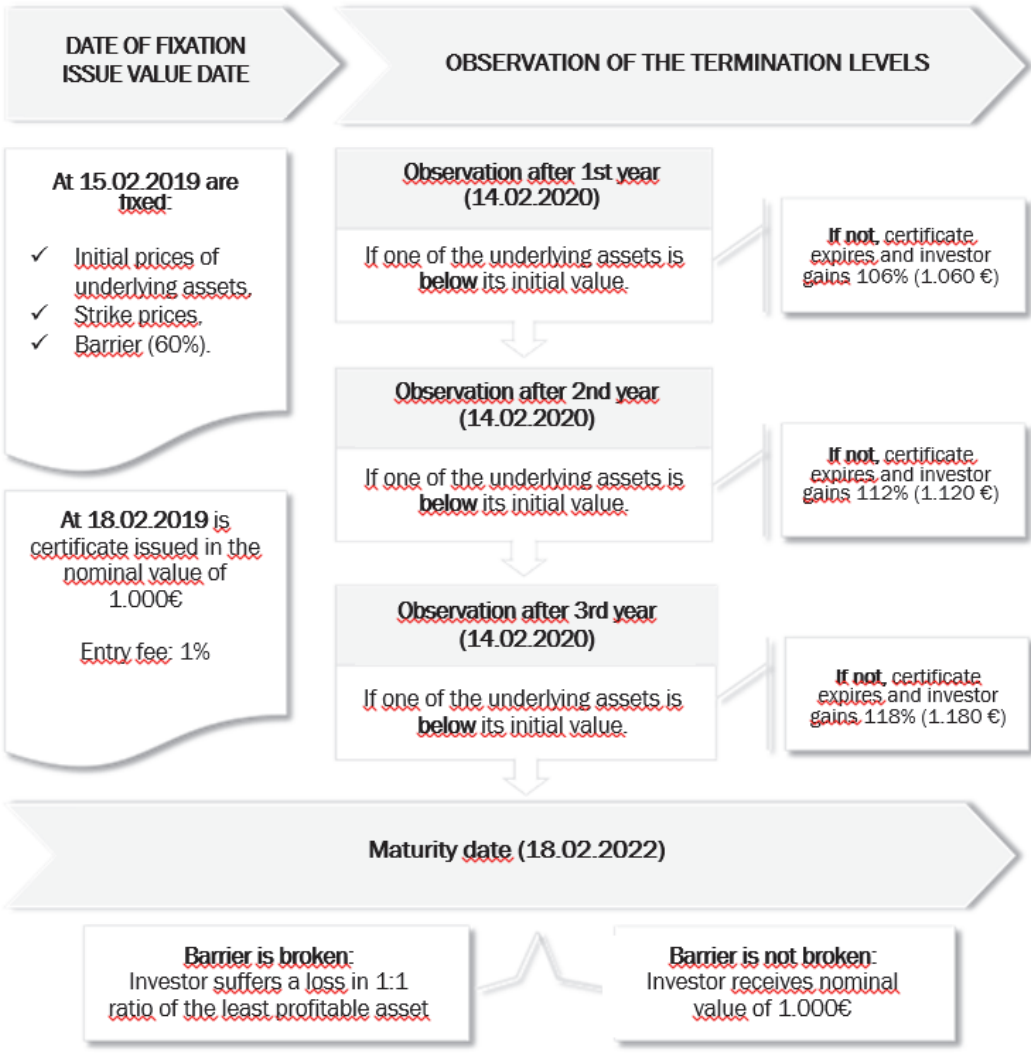

Figure 1. Scheme of the proposed 3-year "SanNes Express" certificate

Source: own design

Depending on the performance of the underlying assets, only one of three options may be on the repayment dates:

– *Early repayment:*

If any cash-or-nothing call option is positively exercised, the certificate is redeemed prematurely and expires. The issuer obtains a specified amount of K from the option (60 and 120, respectively 180, depending on the year in which it occurs) and the amount of EUR 1,000 at the maturity of the bond. Then issuer has the necessary amount which has to pay the investor. In theory, the issuer has a profit of EUR 45.81 increased by the potential gain on those options that expire and will be exercised positively later than this situation.

– *The barrier level is unreached at the maturity date:*

Neither of the three cash-or-nothing call options was realized positively, since at least one asset was below the initial value $\left(S_1(\tau) < S_{10} \vee S_2(\tau) < S_{20}\right)$ and the barrier was not reached, ie 120

$B_1 < S_1(\tau) \wedge B_2 < S_2(\tau)$. In this case, the investor received only payment of EUR 1,000 (secured by the bond). In addition, the issuer also has barrier options. If $S_1(\tau) < S_{10}$, then he can buy the asset S_1 in the market at price $S_1(\tau)$ and using the option, can sell at the initial (higher) price of S_{10}. For that reason, he will get additional resources in the amount of the difference $S_{10} - S_1(\tau)$. If $S_1(\tau) > S_{10}$, he let option expires only in case of $S_2(\tau) < S_{20}$. Otherwise the cash-or-nothing option would be realized. He buys a second asset S_2 at the price $S_2(\tau)$ and using the option, can sell at the initial (higher) price of S_{20}. Again, his profit is that difference. For the issuer the ideal situation is if $B_1 < S_1(\tau) < S_{10}$, but also $B_2 < S_2(\tau) < S_{20}$, because his additional profit is the sum of the realization of both barrier options.

The barrier level is reached at the maturity date:

If the barrier level is reached at the maturity date, ie. $(S_1(\tau) < B_1 \vee S_2(\tau) < B_2)$, then the investor suffers a loss and only part of invested amount can be returned based on the performance of the underlying asset. If it is the underlying asset S_1, then the investor receives the amount of Z_1, where $Z_1 = 1000\frac{S_1(\tau)}{S_{10}}$, but if it is the underlying asset S_2, then the investor receives the amount of Z_2, where $Z_2 = 1000\frac{S_2(\tau)}{S_{20}}$. In the case that only one barrier is reached, the investor may exercise option on the second asset and make additional profit.

Following Table 2 presents payment profiles for investor based on performance of the underlying assets in various scenarios.

Table 2. Payment profiles for investor based on performance of the underlying assets

Investor's payment under various conditions:
1,060 EUR in 2020 ... if $S_1 \geq 75.25$ EUR $\wedge$ $S_2 \geq 78.64$ EUR
1,120 EUR in 2021 ... if $S_1 \geq 75.25$ EUR $\wedge$ $S_2 \geq 78.64$ EUR
1,180 EUR in 2022 ... if $S_1 \geq 75.25$ EUR $\wedge$ $S_2 \geq 78.64$ EUR
1,000 EUR in 2022 ... if $S_1 \in (45.15;75.25)$ EUR $\wedge$ $S_2 \in (47.18;78.64)$ EUR
1,000 x $\min\left\{\frac{S_1}{75.25};\frac{S_2}{78.64}\right\}$ EUR in 2022 ... if $S_1 \leq 45.15$ EUR $\vee$ $S_2 \leq 47.18$ EUR

Source: own design

The most ideal situation for an investor is that on the last valuation date (after 3 years), the values of the two underlying assets will be above their starting values. In this case, the investor has returns 6% per year, and the issuer pays him EUR 1,180. However, this means for the issuer that his profit is EUR 45.81. On the other side, the worst situation for an investor is if the barrier is reached by at least one asset. At that time, the investor participates in the loss of the underlying asset with less performance. Of course, issuer set the certificate's barrier level at the issue time.

This situation is most appropriate for the issuer, as his possible profit is at least EUR 645.81 (for case of exactly assets decline of 40%). However, the investor does not consider this situation and believes that there is sufficient buffer for a 40% decline in assets. There is still possibility to increase the attractiveness of the certificate, for example by increasing the annual rate of return to more than 6%, or, for example, decreasing the barrier level to less than 60% of the initial value, thereby increasing the security buffer for the investor. These certificates are suitable for short-term investors who expect the market to move sideways or to increase slightly. At the same time, they assume that the assets do not break the barrier during the life of the product and are therefore willing to take the risk of losing up to the invested amount.

3.1 Sensitivity analysis

On the other hand, there are ways to increase the attractiveness of the certificate to the investor. The issuer has possibility to reduce his profit in order to improve the investor's conditions by changing some parameters, while the basic principles of operation, maturity information and the initial values of the underlying assets fixed on the initial valuation date of the express certificate remain the same. Increase in annual return - for the investor in case of early repayment, while the original proposed certificate provided a 6% annual return. While all other parameters remain the same and increasing the annual return to 8%, the issuer would have the following costs:

- purchase Zero coupon 3-year bond with a 5% discount rate, with a nominal value of EUR 1,000 at the present price EUR 887.70,
- purchase cash-or-nothing call options for possible payouts K=80 on 1 year (T=1) at EUR 17.43,
- purchase cash-or-nothing call options for possible payouts K=160 on 2 year (T=2) at EUR 29.70,
- purchase cash-or-nothing call options for possible payouts K=240 on 1 year (T=3) at EUR 39.07,
- purchase of the European down and knock-out put option with the barrier level 60% on asset S_1 at EUR 4.80,
- purchase of the European down and knock-out put option with the barrier level 60% on asset S_2 at EUR 7.03.

In order issuer would be able to pay a specific return in early prepayment, then parameters of the correlation cash-or-nothing options will be changed. The issuer pays EUR 98.03 for the purchase of all options and, together with the bond purchase at EUR 887.70, his total cost is EUR 985.73. An entrance fee is in the amount of 1% of the nominal value of EUR 1,000. i.e. EUR 10. Therefore, the smallest possible issuer's profit is EUR 24.27, ie. 2.427%. If the issuer is willing to reduce this profit, he could increase the annual return to 9%. This decision is connected with an additional increase in the cost of options (EUR 108.82). Together with the bond purchase, the total cost would be EUR 996.52 and his lowest profit together with the entry fee would be EUR 13.48, ie. 1.348%. In this case, the investor would be able to receive early repayments of EUR 90 in the first year, EUR 180 in the second year and up to EUR 270 in the third year, which would certainly increase his attention when investing money in investment certificates. Following Table 3 presents certificate payment profiles of Alternative no. 1 and 2 for investor based on performance of the underlying assets in various scenarios.

Table 3. Certificate payment profiles for investor based on performance of the underlying assets – Alternative no. 1 and 2

Investor's payment under various conditions:
1,080 ∨ 1,090 EUR in 2020 ... if $S_1 \geq 75.25$ EUR ∧ $S_2 \geq 78.64$ EUR
1,160 ∨ 1,180 EUR in 2021 ... if $S_1 \geq 75.25$ EUR ∧ $S_2 \geq 78.64$ EUR
1,240 ∨ 1,270 EUR in 2022 ... if $S_1 \geq 75.25$ EUR ∧ $S_2 \geq 78.64$ EUR
1,000 EUR in 2022 ..... if $S_1 \in (45.15;75.25)$ EUR ∧ $S_2 \in (47.18;78.64)$ EUR
$1{,}000 \times \min\left\{\frac{S_1}{75.25};\frac{S_2}{78.64}\right\}$ EUR in 2022 ... if $S_1 \leq 45.15$ EUR ∨ $S_2 \leq 47.18$ EUR

Source: own design

However, increasing early repayment payments also increase the issuer's cost for the options purchase, which in the end reducing his minimum profit.

A second alternative is to lower the barrier level to less than 60% of the initial value, which will increase the scope for a possible underlying assets decline. Figure 2 shows the change of the barrier level for Sanofi SA and Nestlé SA.

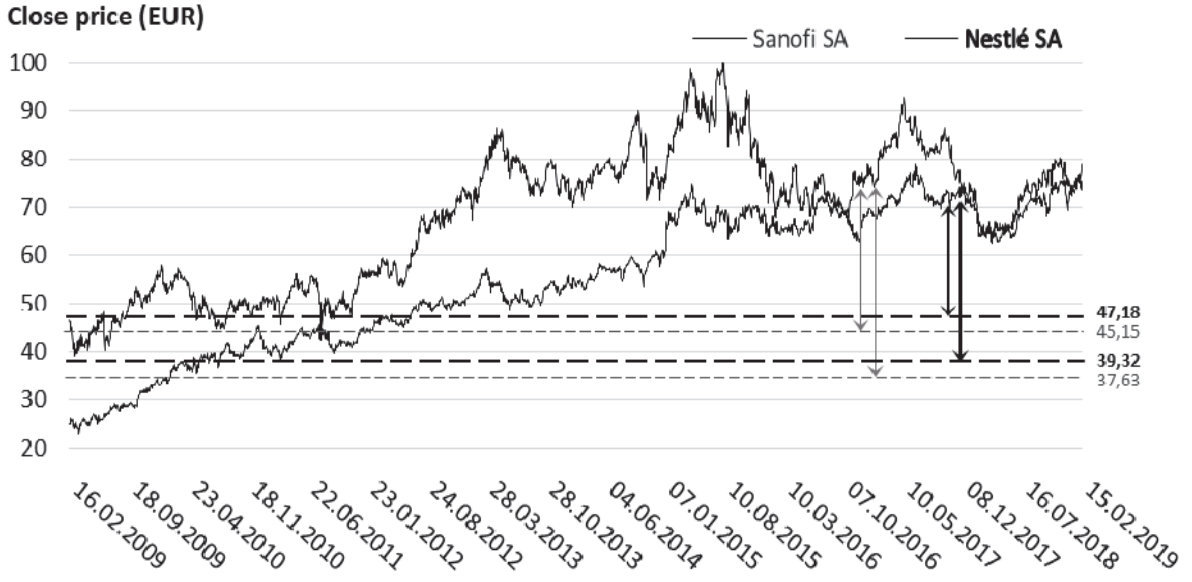

Figure 2. Shift of the barrier level from 60% to 50% of the underlying assets value

Source: own design based on data from Onvista

By lowering the barrier level, it again effects on the issuer's costs, as the barrier options will be more expensive, which increases the cost at the issue time. The certificate is again designed from the same components, but the parameters of the barrier options are changed:

- purchase Zero coupon 3-year bond with a 5% discount rate, with a nominal value of EUR 1,000 at the present price EUR 887.70,
- purchase cash-or-nothing call options for possible payouts K=60 on 1 year (T=1) at EUR 13.08,
- purchase cash-or-nothing call options for possible payouts K=120 on 2 years (T=2) at EUR 22.28,
- purchase cash-or-nothing call options for possible payouts K=180 on 3 years (T=3) at EUR 29.30.
- purchase of the European down and knock-out put option with the barrier level 50% on asset S_1 (the strike price $X_1 = S_{10}$) and the expiration time T = 3 at EUR 8.90,
- purchase of the European down and knock-out put option with the barrier level 50% on asset S_2 (the strike price $X_2 = S_{20}$) and the expiration time T = 3 at EUR 9.42.

The increase cost for barrier options purchase (EUR 18.32) also increases the issuer's total costs to EUR 970.68. This was also reflected in the possible profit of the issuer, whose lowest yield is EUR 39.32, ie. 3.93%. Following Table 4 presents certificate payment profiles of Alternative no. 3 for investor based on performance of the underlying assets in various scenarios.

Table 4. Certificate payment profiles for investor based on performance of the underlying assets – Alternative no. 3

Investor's payment under various conditions:
1,060 EUR in 2020 ... if $S_1 \geq 75.25$ EUR $\wedge$ $S_2 \geq 78.64$ EUR
1,120 EUR in 2021 ... if $S_1 \geq 75.25$ EUR $\wedge$ $S_2 \geq 78.64$ EUR
1,180 EUR in 2022 ... if $S_1 \geq 75.25$ EUR $\wedge$ $S_2 \geq 78.64$ EUR
1,000 EUR in 2022 ... if $S_1 \in (37.63;75.25)$ EUR $\wedge$ $S_2 \in (39.32;78.64)$ EUR
$1{,}000 \text{ x } \min\left\{\frac{S_1}{75.25};\frac{S_2}{78.64}\right\}$ EUR in 2022 ... if $S_1 \leq 37.63$ EUR $\vee$ $S_2 \leq 39.32$ EUR

Source: own design

Although the amount of the investor's payout has not changed, the conditions for two situations on the maturity date have changed. The nominal payout interval of EUR 1.000 has been widened and there has been area for a fall in the underlying assets. However, if this situation were to occur and the fall in at least one asset would be greater than 50%, in this case the issuer has the lowest return of EUR 539.32.

Combination of alternative no. 1 and 3, the investor would be able to earn an annual return of 8% with an area of 50% decline in the underlying assets. The issuer would have the cost for the purchase of correlation options for a payout of EUR 80, EUR 160 and EUR 240 in the amount of EUR 86.20 and for the purchase of barrier options with a 50% barrier level at EUR 18.32. Together with the bond purchase at EUR 887.70, issuer's total costs would be in amount of EUR 992.22 and his guaranteed profit in amount of EUR 17.78, ie. 1,778% of nominal value.

Following Table 5 presents certificate payment profiles of Alternative no.4 for investor based on performance of the underlying assets in various scenarios.

Table 5. Certificate payment profiles for investor based on performance of the underlying assets – Alternative no. 4

Investor's payment under various conditions:
1,080 EUR in 2020 ... if $S_1 \geq 75.25$ EUR $\wedge$ $S_2 \geq 78.64$ EUR
1,160 EUR in 2021 ... if $S_1 \geq 75.25$ EUR $\wedge$ $S_2 \geq 78.64$ EUR
1,240 EUR in 2022 ... if $S_1 \geq 75.25$ EUR $\wedge$ $S_2 \geq 78.64$ EUR
1,000 EUR in 2022 ... if $S_1 \in (37.63;75.25)$ EUR $\wedge$ $S_2 \in (39.32;78.64)$ EUR
$1{,}000 \times \min\left\{\frac{S_1}{75.25};\frac{S_2}{78.64}\right\}$ EUR in 2022 ... if $S_1 \leq 37.63$ EUR $\vee$ $S_2 \leq 39.32$ EUR

Source: own design

In this case, product will give the investor a higher yield (the largest at 24%) and a larger security buffer (down of the underlying assets up to 50%).

3.2 Discussion

Due to the growing interest of investors in express certificates because of their attractive structure, especially in terms of potential annual returns and at the same time a certain amount of security buffer, we designed an express certificate for two specific underlying assets. It is a three-year certificate providing a 6% return and allowing a decrease in assets up to 40%, with the mentioned possible alternatives to increase the attractiveness for the investor. Altogether, 5 certificates are proposed, the key variable parameters are: annual yield and barrier level. By analyzing possible scenarios depending on the performance of the underlying assets, such as Sanofi and Nestlé, we have described the worst and most ideal situation for both the investor and the issuer for each option. The issuer, unlike the investor, achieves a certain profit in every situation, which varies depending on the set parameters of the product. Following Table 6 shows an overview of proposed express certificates. An analytical expressions of the profit function for individual options that create the certificates are shown in Attachment (Table 7).

Table 6. Overview of express certificate alternatives in terms of issuer and investor profits

	Parameters	*The issuer's net profit*	*The issuer's profit in an ideal situation*	*The investor's profit in an ideal situation*	*Potential loss*
Basic certificate	6% annual yield barrier level 60%	4,581 % 45,81 EUR	≥ 645,81 EUR	18 % 1.180 EUR	YES if drop ≥ 40 %
Alternative 1	8 % annual yield barrier level 60%	2,427 % 24,27 EUR	≥ 627,27 EUR	24 % 1.240 EUR	YES if drop ≥ 40 %
Alternative 2	9% annual yield barrier level 60%	1,348 % 13,48 EUR	≥ 613,48 EUR	27 % 1.270 EUR	YES if drop ≥ 40 %
Alternative 3	6% annual yield barrier level 50%	3,932 % 39,32 EUR	≥ 539,32 EUR	18 % 1.180 EUR	YES if drop ≥ 50 %
Alternative 4	8% annual yield	1,778 % 17,78 EUR	≥ 517,78 EUR	24 % 1.240 EUR	YES if drop ≥ 50 %

Source: own design

Within the “Express SanNes” certificate, the alternative no. 2, is an interesting possibility to invest money. This alternative provides a nine per cent annual return, which in an ideal situation can make a profit of 27%, but in the case of a potential loss if the asset falls by 40% or more at maturity date. If the investor is more conservative and demands a larger safety buffer for a fall, alternative no. 4 is the best option. It provides capital protection of up to a 40% price drop in assets with still attractive possible yield of 24%.

In terms of risk, the investor is not exposed to currency risk for the proposed certificates, as both the underlying assets and the payouts are denominated in the same currency. As the basic express certificate only provides partial capital protection, an investor faces market risk if the barrier is broken through at least one of the underlying assets in maturity date. Credit risk is related to the creditworthiness of the issuer, as rated by credit rating agencies, but the issuers of such products are primarily larger and reputable banks.

In addition, there are other possibilities for certificate modifications and combinations. In addition to the annual return rate of and barrier level, it is possible to change, for example the maturity, the type of underlying, or the level of early repayment, which may not be the same in each year, but may decrease or increase by a certain percentage. It is also possible to change the construction elements. The issuer may purchase the annual correlation options in each year, respectively may use other correlation options in combination with others. He may also purchase specific underlying assets instead of purchasing a zero coupon bond. In the case of shares, the issuer may even use dividends paid, for example, to pay options. There are many possibilities and therefore there is not only one strategy used to construct specific types of certificate.

However, the proposed certificates also provide attractive returns and partial capital protection in the form of a safety buffer in the short term. Therefore, these certificates could be interesting and the investor could choose based on his investment profile and use his free funds for appreciation.

CONCLUSION

In the financial markets, investors are increasingly interested in alternative investment ways to traditional financial instruments, such as structured products, in the largest proportion of investment certificates. They complement the gap between investment in bonds that offer a certain return at low risk and investment in equities that, while yielding higher returns, have a higher risk. Therefore they extend the possibilities of appreciation of financial resources while allowing investors to obtain an interesting and specific profile of profitability and risk. As a result, their structure is more complex and wider knowledge is needed to better understand the matter. Even a study from 2018 showed that more informed and qualified investors are investing in more complex products.

This paper is focused on the creation of the modification of express certificates as the new tool for financing of issuers and investment for investors. Design of these products is expressed through the European two-asset correlation options and barrier options. For understanding of investment certificates' design, it was introduced the review of the literature dealing with the structured products. On the basis of the existing empirical studies, the scientific problem of the paper is to design and demonstrate the nature of the modification of express certificates creation through financial engineering. The approach is based on the pricing models of using two-asset correlation options and barrier options. Our empirical approach is applied on the basket stocks in Sanofi S.A. and Nestle S.A with the issue date 18.02.2019. Due to lack of two-asset correlation option we calculated their prices according to Haug model.

The main contribution of this work is the combination of theoretical knowledge about the investment certificates as a group of structured products, knowledge of the use of correlation and

barrier options, which aims to contribute to their better understanding. Another benefit is the presentation of the proposed structured product on two underlying assets and its potential to increase attractiveness for the investor. According to the methodological point of view, our approach can provide as an inspiration for creation of the further types of the investment certificates.

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ATTACHMENTS

Table 7. Payout profiles in the analytical expression of the options applied to the real underlying assets of each alternative of the proposed express certificate

"Express SanNes" certificate - basic

$$P_{CoN\ call}=\begin{cases}0 & if\ S_1(2020)\le 75.25 \vee S_2(2020)\le 78.64\\ 60 & if\ S_1(2020)>75.25 \wedge S_2(2020)>78.64\end{cases}$$

$$P_{CoN\ call}=\begin{cases}0 & if\ S_1(2021)\le 75.25 \vee S_2(2021)\le 78.64\\ 120 & if\ S_1(2021)>75.25 \wedge S_2(2021)>78.64\end{cases}$$

$$P_{CoN\ call}=\begin{cases}0 & if\ S_1(2022)\le 75.25 \vee S_2(2022)\le 78.64\\ 180 & if\ S_1(2022)>75.25 \wedge S_2(2022)>78.64\end{cases}$$

$$P_{DO\ put}=\begin{cases}0 & if\ S_1(2022)\le 45.15\\ 75.25-S_1(2022) & if\ S_1(2022)>45.15 \wedge S_2(2022)>45.15\\ 0 & if\ S_1(2022)>45.15 \wedge S_2(2022)>45.15\end{cases}$$

$$P_{DO\ put}=\begin{cases}0 & if\ S_2(2022)\le 47.18\\ 78.64-S_1(2022) & if\ S_2(2022)>47.18 \wedge S_1(2022)<47.18\\ 0 & if\ S_2(2022)>47.18 \wedge S_1(2022)\ge 47.18\end{cases}$$

"Express SanNes" certificate – Alternative no. 1

$$P_{CoN\ call}=\begin{cases}0 & if\ S_1(2020)\le 75.25 \vee S_2(2020)\le 78.64\\ 80 & if\ S_1(2020)>75.25 \wedge S_2(2020)>78.64\end{cases}$$

$$P_{CoN\ call}=\begin{cases}0 & if\ S_1(2021)\le 75.25 \vee S_2(2021)\le 78.64\\ 160 & if\ S_1(2021)>75.25 \wedge S_2(2021)>78.64\end{cases}$$

$$P_{CoN\ call}=\begin{cases}0 & if\ S_1(2022)\le 75.25 \vee S_2(2022)\le 78.64\\ 240 & if\ S_1(2022)>75.25 \wedge S_2(2022)>78.64\end{cases}$$

$$P_{DO\ put}=\begin{cases}0 & if\ S_1(2022)\le 45.15\\ 75.25-S_1(2022) & if\ S_1(2022)>45.15 \wedge S_2(2022)<45.15\\ 0 & if\ S_1(2022)>45.15 \wedge S_2(2022)\ge 45.15\end{cases}$$

$$P_{DO\ put}=\begin{cases}0 & if\ S_2(2022)\le 47.18\\ 78.64-S_1(2022) & if\ S_2(2022)>47.18 \wedge S_1(2022)<47.18\\ 0 & if\ S_2(2022)>47.18 \wedge S_1(2022)\ge 47.18\end{cases}$$

Express SanNes" certificate – Alternative no. 2

$$P_{CoN\ call}=\begin{cases}0 & if\ S_1(2020)\le 75.25 \vee S_2(2020)\le 78.64\\ 90 & if\ S_1(2020)>75.25 \wedge S_2(2020)>78.64\end{cases}$$

$$P_{CoN\ call}=\begin{cases}0 & if\ S_1(2021)\le 75.25 \vee S_2(2021)\le 78.64\\ 180 & if\ S_1(2021)>75.25 \wedge S_2(2021)>78.64\end{cases}$$

$$P_{CoN\ call}=\begin{cases}0 & if\ S_1(2022)\le 75.25 \vee S_2(2022)\le 78.64\\ 270 & if\ S_1(2022)>75.25 \wedge S_2(2022)>78.64\end{cases}$$

$$P_{DO\ put}=\begin{cases}0 & if\ S_1(2022)\le 45.15\\ 75.25-S_1(2022) & if\ S_1(2022)>45.15 \wedge S_2(2022)<45.15\\ 0 & if\ S_1(2022)>45.15 \wedge S_2(2022)\ge 45.15\end{cases}$$

$$P_{DO\ put}=\begin{cases}0 & if\ S_2(2022)\le 47.18\\ 78.64-S_1(2022) & if\ S_2(2022)>47.18 \wedge S_1(2022)<47.18\\ 0 & if\ S_2(2022)>47.18 \wedge S_1(2022)\ge 47.18\end{cases}$$

Express SanNes" certificate – Alternative no. 3	*Express SanNes" certificate – Alternative no. 4*
$P_{CoN\ call}=\begin{cases}0 & \text{if } S_1(2020)\le 75.25 \vee S_2(2020)\le 78.64\\ 60 & \text{if } S_1(2020)>75.25 \wedge S_2(2020)>78.64\end{cases}$	$P_{CoN\ call}=\begin{cases}0 & \text{if } S_1(2020)\le 75.25 \vee S_2(2020)\le 78.64\\ 80 & \text{if } S_1(2020)>75.25 \wedge S_2(2020)>78.64\end{cases}$
$P_{CoN\ call}=\begin{cases}0 & \text{if } S_1(2021)\le 75.25 \vee S_2(2021)\le 78.64\\ 120 & \text{if } S_1(2021)>75.25 \wedge S_2(2021)>78.64\end{cases}$	$P_{CoN\ call}=\begin{cases}0 & \text{if } S_1(2021)\le 75.25 \vee S_2(2021)\le 78.64\\ 160 & \text{if } S_1(2021)>75.25 \wedge S_2(2021)>78.64\end{cases}$
$P_{CoN\ call}=\begin{cases}0 & \text{if } S_1(2022)\le 75.25 \vee S_2(2022)\le 78.64\\ 180 & \text{if } S_1(2022)>75.25 \wedge S_2(2022)>78.64\end{cases}$	$P_{CoN\ call}=\begin{cases}0 & \text{if } S_1(2022)\le 75.25 \vee S_2(2022)\le 78.64\\ 240 & \text{if } S_1(2022)>75.25 \wedge S_2(2022)>78.64\end{cases}$
$P_{DO\ put}=\begin{cases}0 & \text{if } S_1(2022)\le 37.63\\ 75.25-S_1(2022) & \text{if } S_1(2022)>37.63 \wedge S_2(2022)<37.63\\ 0 & \text{if } S_1(2022)>37.63 \wedge S_2(2022)\ge 37.63\end{cases}$	$P_{DO\ put}=\begin{cases}0 & \text{if } S_1(2022)\le 37.63\\ 75.25-S_1(2022) & \text{if } S_1(2022)>37.63 \wedge S_2(2022)<37.63\\ 0 & \text{if } S_1(2022)>37.63 \wedge S_2(2022)\ge 37.63\end{cases}$
$P_{DO\ put}=\begin{cases}0 & \text{if } S_2(2022)\le 39.32\\ 78.64-S_1(2022) & \text{if } S_2(2022)>39.32 \wedge S_1(2022)<39.32\\ 0 & \text{if } S_2(2022)>39.32 \wedge S_1(2022)\ge 39.32\end{cases}$	$P_{DO\ put}=\begin{cases}0 & \text{if } S_2(2022)\le 39.32\\ 78.64-S_1(2022) & \text{if } S_2(2022)>39.32 \wedge S_1(2022)<39.32\\ 0 & \text{if } S_2(2022)>39.32 \wedge S_1(2022)\ge 39.32\end{cases}$

Notes: CoN call – correlation options cash-or-nothing, DO put – barrier options down and knock-out, S_1 – Sanofi SA, S_2 – Nestlé SA