APPLICATION OF THE METHODOLOGY CALLED "ARTIFICIAL SOCIETIES"

PRIMJENA METODOLOGIJE POZNATE KAO "VJEŠTAČKA DRUŠTVA »

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Abstract: *The methodology of artificial societies, coming from the sophisticated development of so called agent based models, is going to be breakthrough in social sciences. In the paper I try to show it on the examples of mathematical economics and related topics. The growing complexity of mathematical models can be overcome by construction of artificial worlds and following computer experiments. The first non trivial results are in place. Some intriguing questions and problems are formulated, answer on which one can expect under application of the new methodology.*
Key words: *matematical model, artificial societies, agent, virtual words, methodology*

Apstrakt. *Metodologija vještačkih društava, koja ima polazište u sofisticiranom razvoju tzv. modela zasnovanih na agentu, predstavlja napredak u društvenim naukama. U ovom radu to pokušavamo prikazati na primjerima matematičke ekonomije i s njom povezanih tema. Narasla kompleksnost matematičkih modela može biti prevaziđena konstukcijom vještačkog svijeta i korišćenjem kompjuterskih eksperimenata. Prvi, nimalo trivijalan rezultat, je ostvaren. Neka intrigirajuća pitanja i problemi su formulisani, odgovor na njih može da se dobije primjenom ove nove metodologije.*
Ključne riječi : *matematički model, vještačka društva, agent, virtuelni svijet, metodologija*

JEL Classification numbers: B41, C02, C15, C90, H41.
Original scientific paper; Recived: July 20, 2007.

Where mathematical modeling faces a limit.

Mathematical modeling of a real world is a practically universal methodology. One considers serious or modern theory if it based on mathematical model. It is easy to list numerous examples of these theories in any field of research. The economic science is not an exception. Let me mentioned for example widely used general economic equilibrium theory, which one can consider as a part of theory of games. Taking leading economic journals one can see a lot of mathematics.

But it is clear that mathematical modeling has its limits. Reality is too complicated to be modeled as precisely as one wants to have. Making more realistic mathematical models we produce monsters, which it is impossible to analyze by standard methods of logics.

Economic science is developing to the direction of getting closer to natural sciences. Now it is possible to make experiments in the sphere of economics, what was difficult to imagine a century ago. The first breakthrough was done when laboratory economics appeared after Vernon Smith's methodology and related activity. The next crucial step is connected with the quick development of virtual worlds and artificial societies. One can raise much broader set of questions, which are impossible to formulate in mathematical terms.

Let me illustrate the thesis by the mathematical model of an economy, which is a combination of the well-known economic models.

In the modern economic literature one uses a notion of an (economic) agent. The agent is autonomous to make its decisions. The standard classification of the decisions is the following.

1d. An agent chooses a so-called consumption basket of all types of goods and services.
2d. An agent chooses a jurisdiction to live.
3d. An agent chooses profession and a firm to work.

4d. An agent votes for (a) a political party, (b) economic policy, (c) providing of a certain bundle of public goods, (d) creation or modification of jurisdictions.
5d. An agent decides to participate in clubs, including such club as family.
6d. An agent chooses investment portfolio.

The listed decisions of all agents define a total demand in the society.

The total supply of the society one can define by the following actions and mechanisms.

1s. Production of goods and services by institutions.
2s. A mechanism of jurisdiction's creation and modification.
3s. Professions and jobs.
4s. Political system, type of democracy.
5s. Clubs' formation.
6s. Investment policy.

The basic approach of a main stream's theory consists of defining a kind of equilibrium and finding properties of the equilibrium. The economics literature is full of brilliant models equalizing demand and supply for every point, indicating in the above list.

For example, the point (1d – 1s) gives the very famous Arrow – Debreu model of the general equilibrium, which is classic. The theorem of an equilibrium existence is based on Kakutani fixed point's theorem, optimality of the equilibrium yields from Kuhn – Tacker separation theorem, the proof of finite number of equilibrium states uses some facts of algebraic geometry. The point (2d – 2s) leads to the Tiebout model, (Tiebout Ch. (1956)), which is so popular now that came to the top of topics in terms of citations. The technique to prove equilibrium's existence of theorem is very sophisticated, see Caplin and Neilbuff . The point (4d – 4s) generates a number of relatively simple graceful models, most of them the reader can find in Person and G. Tabellini.

The same situation one sees in the other points.

But if you try to combine all these points into one model you create a monster from the mathematical point of view. It is absolutely impossible to prove something putting all the notions into one wrap. There are a number of attempts to combine just some of the listed points, two or three, to obtain substantial results. No results to be mentioned as visible.

What is an artificial society?

Because of computers there is a natural way to overcome complexity of mathematical models. A computer model can be as complex as possible. There are no near limits, just a computer's memory and human efforts. One calls a computer model of real society as the artificial society.

An artificial society consists of *agents,* which act in *an environment*, following given *rules*. According to the standard terminology the artificial society is a particular case of, so-called, *agent – based models*. Agent – based model are widely used in a number of fields for analysis of problems like, for example, transportation flows, city design, mass services, etc. So, the agent – based model is not necessarily should consider as an artificial society.

Agents in an artificial society perform relatively autonomously. They *make decisions, act* and *interact* with other agents. Agents make decisions as a reaction on the environment and actions of other agents. The key word is an *interaction* between agents. One calls it as social ability of agents and therefore we can talk about an artificial society, as a particular case of agent – based models.

Historically the first example of artificial society was Neumann's cellular automates. The simplest society of cellular automates was proposed by Conway (see Conway J. (1970)), called "Life". The game "Life" looks very simple according to the rules of behavior of agents (black cells). Nevertheless the cellular automata shed new light on the problems, which it is difficult to understand by other methods. See, for example, Hu Bin and Debing Zhang (2006). To the paper we will return below.

I don't go to the history in details. One can find it in the seminal book of Epstein Joshua M. and Axtell Robert (1996) "Growing Artificial Societies". The authors develop the methodology of Artificial Societies, based on relatively simple SugarScape model, which became very popular thanks to the book. The SugarScape model is really very simple, even in comparison with a set of finite automata of von Neumann or Cetlin type, see Цетлин М.Л. (1969). Nevertheless one can see the richness of questions and problems to be raised, discussed and experimented in terms of the SugarScape model.

By the way Epstein and Axtell compare cellular automata with environment or space in their sugarscape model. Under this vision the sugarscape model is the agents traveling among cellular automata.

The SugarScape model.

Let me start with the original version of the model.

At any moment t there is finite number of agents located in the space.

Space is a two dimensional lattice of equal cells.

At any moment t each cell (x,y) has: (1) an agent a $(a_t(x,y) = a)$, if agent a is located in the cell (x,y), or no agents otherwise $(a_t(x,y) = empty\ set)$.

(2) quantity $r_t(x,y)$ of "sugar".

An agent is born with two parameters: vision (number of cells in the lattice to look around) and level of metabolism (quantity of sugar to eat per unit of time to survive). An agent can carry any amount of sugar. An agent dies if it has not enough sugar to eat. The authors explore a number of rules how an agent can be born. For example, simultaneously with its death a new agent is born with randomly choused parameters and location. So the total population of agents stays constant in this case.

Rules for agents.

- Look out in the four (eight) lattice directions and identify the unoccupied site having the most quantity of sugar.
- Move this site and collect all the sugar at this new position.

Based on the simple version of the SugarScape model authors produced a number of experimental calculations and received results, which are consistent with common understanding of human society's features. It looks wonderful, because in the version there are no interactions between agents, which one treats as a basic property of human society.

For example, there is a fundamental problem, which challenged many thinkers of all times. It is distribution of wealth among members of society. The distribution is always quite uneven in any existent society. And the problem is strengthening nowadays because inequality is growing as among people as among countries. The authors of the book find that "under a greate variety of conditions the distribution of wealth on the sugarscape is highly skewed, with most agents having little wealth."

The agents compete to each other for the sugar and the macro structural result is a concentration of population in sugar – rich arias. Even in this simple version of the Sugarscape model without interactions between agents, authors explore a way, how to avoid the concentration of population, particularly with phenomena of pollution.

The pollution can be introduced as an impact of collection and eating of sugar. So each cell contains sugar and the level of pollution. An agent moves to a free sell with maximum sugar to pollution ratio. The rule changes the picture dramatically. First of all the number of deaths increases and second, distribution of population across territory becomes more even.

In advanced versions of the Sugarscape model different types of interaction between agents and some complexities are introduced. In allows analyzing a great variety of phenomena, taking place in human societies. Sex, Culture, Conflicts, Decease, Heritage and a number of others properties of human society can be investigated in terms of the Sugarscape model.

Especially interesting findings coming from the Sugarscape model one can see in sphere of economics. The authors make the model more complicated introducing the second good "Spice".

In the Sugarscape model with two commodities the agents' rules of behavior are getting more complicated. An agent moves to the neighboring empty sell, where augmented welfare is maximal. The welfare function of an agent is defined on the two goods. So, in this respect the behavior of agents looks similar to their behavior in the case of one good. But an agent has new possibility to exchange by commodities with another agent, if it would be beneficial for both.

Under condition of finite time horizon one can formulate the Sugarscape model as a standard Arrow – Debreu model, basic one in the general equilibrium theory. Namely, an agent a has the initial endowment $w_{a,t}(x,y)$, where for every t and given (x,y) pair $w_{a,t}(x,y)$ shows a quantity of sugar and spice, (x,y) is the cell of its location at the moment $1,\ t = 1,2,\dots,T$. Then it is possible to compare an equilibrium state of the Arrow – Debreu model and the results of calculations for the Sugarscape model. The author of the book made a lot of calculations, starting from different initial locations of agents. Always mean price of exchanges of sugar and spice between agents tends to the equilibrium price. But final position of agents depends on their initial location. In particular it means that the two absolutely identical agents with the same initial endowment can be in different positions finally. It confirms once more the unevenness of distribution of wealth, independently on the variety of initial conditions and variety of genetic characteristics of agents.

In this connection the remark of the authors about principal difference between the equilibrium in the sense of general equilibrium theory and the equilibrium (called statistical equilibrium) of the Sugarscape model looks a little bit strange. They mentioned that according to the general equilibrium theory identical agents with equal endowment have equal behavior and equal meaning of the utility function in an equilibrium state. It is correct. But after that they state that in the Sugarscape model identical agents with the same endowment finally come to very different positions. To be exact, in the Sugarscape model agents have different endowment, because into definition of the endowment it is necessary to include its location.

Collective goods' economics.

The methodology of "Artificial Societies" is perfectly fitted to the economics of collective goods. The collective goods are products of interactions or mutual actions of agents, by definition. The notion of collective goods appeared relatively recently. Pioneering works in the field belong to the two Nobel prize winners in Economics Samuelson and Buchanan. See, Samuelson, P. A. (1954), Buchanan, J. M. (1965).

The process of production, distribution and consumption of collective goods is collective by definition. It does not regulated by market, although for production it is necessary to use private goods operated on a standard market. The volume of tangible and intangible collective goods, which turned out in the modern economics, is growing and is now more then one/third of GDP by share in most countries. Needless to say, that these estimations are very conditional because collective goods are not measured in market terms. Sometimes the cost of production of collective goods is huge, like in the case of infrastructure, sometimes – close to zero, like in the case of marriage.

One of the basic problems in the economics of collective goods is a group formation. Below I show the formation of pairs to produce and consume a collective good, one can call "share time". Details see in Макаров В. Л. (2007).

So, there is an artificial society of agents where the problem of every agent is to split its time onto two parts. The first part's time uses to produce a private good and the rest of time spends with a partner, whom it is necessary to find. The partner should agree to spend exactly the same piece of time, which the agent wants to.

Notations.

N – total number of agents;
i - number of an agent;
w_i – total reserve of time, the agent has;
$(w_i - c_i)$ - quantity of private good, produced by the agent;
x_i - quantity of private good, consumed by the agent;
c_i - time of the agent, which it spends together with a partner;
a_i - level on propensity to be alone;
$u_i(x_i, c_i) = a_i* x_i + x_i* c_i$ - utility function og the agent;
$d(i,j)$ - characteristics function defined on pairs (i,j) with additional condition that if $d(i,j)=1$, then $d(i,k)=0$, $dk,j)=0$ for all others k;

<u>Social planner's problem.</u>

As usual it makes sense to take understandable enough criteria of optimality, to have possibility to compare different approaches and calculations. Here the criteria is the sun of all individual utility functions. After that one can formulate the optimization problem to find maximum this sum of utility functions of all agents.

Namely,

To find: $\{(x_i, c_i)\}$, $i = 1,2,\ldots,N$; $d(i,j)$, $i = 1,2,\ldots,N$, $j = 1,2,\ldots,N$, $i = j$, such, that
$x_i = w_i - c_i$
$c_i = c_j$, if $d(i,j)=1$,
$c_i = c_j = 0$, if $d(i,j)=0$,
$sum_i(u_i(x_i, c_i))$ achieves maximal value.

` Every agent tries to choose the partner, who agrees for interaction with minimal deviation (measured by meaning of utility function) from optimal c. Thus the agent makes an order for all other agent accordance the parameter. The real interaction (deal) happens, when both participants agree.

<u>Rules of interactions between agents.</u>

Every agent calculates best for herself period of time to be together. Namely she solves the problem of the function $a_i* x_i + x_i* c_i$ maximization over variable c_i, where $x_i = w_i - c_i$. The first order conditions give $c_i = (w_i - a_i)/2$. Consider the value c_i as desirable.

After that given (for example, randomly chosen) agent looks at all agents in her neighborhood. The neighborhood is defined by the ability of the agent to see around (length of horizon). She make a proposal to the agent, whose desirable meaning is maximally close to her desirable meaning. The chosen agent accepts the proposal if there are no other more desirable ones in her neighborhood. The period of time, which agreed agents spend together, is equal to arithmetical average from the two desirable periods. In the case when the deal is not made, the second agent makes a proposal to her desirable agent around. The process is finished when there are no agents who want to make a proposal. The natural question appears: comes the described process to optimal solution of the social planner's problem or not?

The answer on the question one can receive by making calculations according to the methodology of "artificial societies". We made a number of the calculations, where the space was two - dimensional lattice. It is clear that in principle the process does not come to the maximum (to the solution of the social planner problem). But it is interesting to know the dependence of the meaning of the criteria on the parameters, initial conditions and random details.

We limited ourselves by the case where all agents have the same reserve of time and different propensities to be alone. No essential surprises were discovered. The results are consistent with intuition. For example, longer horizon to look around - closer to maximum, all other conditions are equal. Longer horizon, less dependence on the initial location of agents. Practically maximum was never reached. It is also understandable, because the arithmetical average is not ideal solution of the deal between two agents. The other more sophisticated rule consists of maximization of the sum of the two individual values, which does not concede with arithmetical average.

Collective behavior of agents (continued).

There are a big variety of the collective behavior types. Staying in the language of simple cellular automata it is possible to study the relatively complex group behavior. The paper Hu Bin and Debing Zhang (2007) brilliantly demonstrate, how the world of cellular automata can be used for explanation of group behavior focused on the loyalty to a group (firm) of its members. The member of the group can have different orientations, for example with motivation to earn money (Economic Being) or motivation to feel comfortable within the group Social Being).

There is the lattice of the finite number of cells. Each cell represents one member of the group.

The loyalty is measured by the three levels: "high", "normal" and "low"

From here a group can be divided further according to two generalized sets of characteristics, and which result in differing motivations: One group being "economic", and the other being "social". According to the theory on employee behaviors", Economic Beings are motivated by material and monetary gains. Social Beings are motivated by social status and the respect of their colleagues.

Competition between political parties.

As I said above, the attempts to take into consideration many factors come to a model, which is difficult to study by a mathematical method. In the section I illustrate the thesis by the important example of the very famous Tiebout model. See, Tiebout Ch. (1956).

The Tiebout model deals with people located in finite number of jurisdictions (regions, towns, etc). Every citizen makes a decision to stay in the jurisdiction where he/she lives or to move to another jurisdiction to maximize her utility function's value. The decision is based on the following information about jurisdictions: a package of local public goods, provided by the jurisdiction and level of taxes in it. In the wide literature dedicated to the Tiebout model one can find numerous results about existence of equilibrium and optimal properties of the equilibrium states.

The situations is getting much more complicated when you incorporate into the model political process of elections. Every political party wants to win elections, and hence to attract as many votes as possible. A formalization of the political process leads to a modification of the Tiebout model, which looks like an artificial society. In the modification agents make the following actions. They move from one jurisdiction to another and they vote for one or another political party or vote for an offered issue in the case of direct democracy.

So, let me formulate the modification based on the paper Collman, Ken, John H. Miller and Scott E. Page (1997) and Данков А. Н. и Макаров В. Л. (2002).

So, there are N inhabitances, each lives in one of the jurisdictions $j = 1,2,\ldots,G$.

The political process is represented by n political parties, which operate in all jurisdictions. Each party k $(k = 1,2,\ldots,n)$ works out and offers to voters its platform $p_k = (p_{ki})_{i=1,2,\ldots,I}$. The platform does not depend on the jurisdiction and formally is I – dimensional Boolean vector, where $p_{ki} = 1$, if the party stays **for** an issue *i* and $p_{ki} = 0$, if the party agitates against the issue. A meaning of the issue in the list may be quite different, like a war in Chechnya, taxes on property or ban of producing human clones.

Every person has her own opinion on all the issues. The opinion is expressed be a number , which may be positive or negative. Let $v_{ai.}$ be the number. The opinions v_{ai} are uniformly distributed on the interval [-1,+1].

Knowing v_{ai}, one can calculate a utility of an agent *a*, if the platform p_k comes true.

Namely,

$$U_a(p_k) = sum_i \, (p_{ki} * v_{ai})$$

An agent votes under various political systems. She votes for the party, which offers a platform with maximal utility for her. Or she votes for the issue, which gives maximal utility.

The world political practice shows different ways of converting individual votes into political decisions. Simplest way is called *referendum* and direct democracy. The result of referendum is a policy (platform) $p= (p_i)$, where quantity of people with $v_{ai}>0$ is greater then quantity of people with $v_{ai}<0$ for all *i*. So, in the case of direct democracy there is no room for political party. People vote not for a party but for issues.

Next one is a form of representative democracy called *direct competition*. In practice direct competition forms *presidential power* under the rule "the winner receive everything". Formally in our term it means that people vote for parties and the party with maximum votes takes an office.

The third one, we take into consideration, is a proportional representation. It is another form of representative democracy called *a parliamentary power*. Every political party, which took part in the election, receives an influence proportional to the number of votes for the party.

Questions, we want to receive the answers.

1. What regime is better (in terms of the total welfare function) and under what conditions?
2. What influence of number of political parties on the value of the welfare function?
3. What influence of number of jurisdictions on the value of the welfare function?
4. What type of party's behavior is better in terms of welfare function and in terms of probability to win or to pick up greater number of votes?

Of course, the number of questions to be raised is much greater. Nevertheless one can propose, that purely mathematical approach will fail to answer the mentioned questions.

So, the behavior of agents is described above. They vote and move if necessary. The behavior of political parties is more complicated. A political party has its platform, which lies in the base of the party. But a party can change its platform to attract more voters. So it is necessary to define the neighborhood of the platform, inside which the party keeps its base. There are more flexible parties then others, depending on the size the neighborhood of its platforms.

It is clear, that one can organize the process of simulations by different ways. One of the ways is the following.

1. One starts with defining the initial conditions: a realization of random distribution of agents between jurisdictions and random distribution of opinions (**v_{ai}**) among agents.
2. The platforms of all political parties are given.
3. Every party tries to find correction of its platform, which brings greater votes. For that the party chooses so called focus group (randomly chosen given number of people). The focus group shows, which policy in the neighborhood gives maximal number of votes.
4. The population votes for the presented programs.
5. Points 3 and 4 can be repeated several times.
6. The results of the election are presented to the population. In means that in the case of referendum it is indicated winning issues in the each jurisdiction. Presidential election gives policies of wining party in the each jurisdiction. And in the case of parliamentary election the each jurisdiction calculates the policy, which should be realized.
7. The agents move to preferred jurisdictions, knowing situation in all jurisdictions.
8. The process begins again from the point 3 and is repeated a given number of times.

The final distribution of the population between jurisdictions and the meaning of the welfare function compared with the solution of the social planner's problem.

Detailed results of the simulations the one can find in Данков А. Н. и Макаров В. Л. (2002).

Here I mention the two most interesting ones. The calculations show that in the case of one jurisdiction presidential regime gives better results in terms of welfare function when the number of parties is lower. If the number of parties is much greater then two, the parliamentary regime is better. In the case of multiple jurisdictions both regimes gives better results under several jurisdictions in comparison with one.

Needless to say, that the results are rather qualitative. One can calculate more precise outcome trying to obtain optimal number of parties and jurisdictions under different regimes.

Artificial Societies and Virtual Worlds.

One can find the evident connection between Artificial Societies and Virtual Worlds. In some sense we can consider an artificial society as a certain type of a virtual world. So the virtual world is a broader concept with less definite boundaries.

Principal feature of the virtual world is an involvement of one or more participants in the experiments. One objects the statement by giving the famous example of the "Star Wars", where there is no involvement of a spectacular into the move's running. But here we have rather an exception then the rule. Nevertheless the example "Star Wars" shows, that it is possible to obtain new scientific results by this instrument. Namely, we can mention the power – law distribution of wealth (in Star Wars and other related games.) as a final outcome of the whole procedure. Indeed it was checked that practically always the end of the war takes place when the distribution of power coincides with the distribution of wealth. Under the condition there is no incentive to continue the war.

Virtual Worlds are not for scientific purposes only as one can see. Moreover they are basically for other purposes.

What for Virtual Worlds?

- Commercial Gaming.
- Socializing / on line community building.
- Education.
- Political Expression Instrument for political debate
- Military Training
- And finally the tools for research, for better understanding of natural worlds.

So we are on the eve of mixture of natural world and virtual one. The virtual world is going to be part of natural one. For a human being it will be difficult to distinguish between natural reality and virtual one. See, for example, the popular movie "Matrix". It can be a source and cause for mental deceases. So, we come to the analogous logical circle: construction of the virtual world, containing inside the virtual world of the, so called, second order. Recall, that the first logical circle one receives in terms of the artificial societies: a perfect artificial society, which is an exact copy of the natural society, must include itself as a proper part.

An ambitious agenda for the future research.

In the terms of artificial societies one meets the opportunity to formulate the problems, the answers to which the mankind is waiting for centuries. Why the formulation in the terms of artificial societies is more promising, then in standard terms? Because

Let us illustrate the statement by a number of examples.

1. One of the puzzled and difficult questions is related to the problem of finite duration of agents' life. Why an agent does not live forever? Why it is more efficient to have the process of birth of new agents, their education, and absorption of knowledge from other agents, instead of accumulation of the knowledge in one agent? So, we are talking about an evolution between agents versus an evolution within an agent.
2. The same question one can ask about the groups of agents. Why is it so big diversity in the duration of life of different types of groups? Some groups live during on generation, like gangs, classmates, busyness' alliances. The others exist for centuries, like empires, nations, religions, etc.
3. In terms of artificial societies one can easily formulate the question: what happened to the society, if the reincarnation takes place. It means that one agent can live number of times, keeping identity. It looks like a mixture between finite and infinite agents' life cases.
4. Why a human being must change a kind of activity during a day, year and life? In this respect the human (animals) society differs from a society of robots.
5. It is easy to imagine a population of agents, where there are no sexes or number of sexes is more then two. What happened then? Is it efficient in some sense in comparison with the traditional population? What are possible advantages?
6. Who knows, may be the instrument of artificial societies will produce better way of future human

development, new models of society and so on. Remember, Leonardo da Vinci used to say that he does not copy a reality but construct it.

7. Other sciences like psychology and philosophy find new arguments for understanding, how emotions, conscience, consciousness, evolve. Neuroscientists recognize six basic emotions: anger, disgust, fear, joy, sadness, and surprise. And the instrument of artificial societies gives more adequate language for definitions of the concepts. It is interesting problem to expand these concepts from individuals to groups. What is memory of the group, what is its consciousness, where the memory of the group is located, and so on?
8. At the end I would like to discuss the urgent problem, which is coming, first of all, to the community of economists as a sound challenge nowadays. It is the problem of happiness, its measurement. The journal "Economist" raised the issue in its publications at last number of year 2006. There is a common statement that a rich country in terms of GDP is not necessarily happiest one. Standard macroeconomic indices like GDP her capita, personal income, individual consumption, general wealth per capita and so on can't serve anymore as basic parameters for measuring prosperity of a country its general success, its superiority over other countries. People want to get life happier rather then richer. A person feels happy if he/she belongs to the top part of the group, he or she considers as very important. It means that the society should be organized in such a way that creates a sufficient number of such groups. As it mentioned by the journal "Economist", in USA there are more then 3000 Halls of Fame. I wrote in Макаров В. Л. (2007) about production of collective goods, which are not included into GDP. One can count it as a first step in the direction of the correction of the standard macroeconomic indices.

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