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Integrated Indicator of the Living Quality Conditions

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ABSTRACT

In this paper, it is proposed a statistical approach to the forma of indices for different directions of the regional socio-econo development. In its application, it is calculated eight indices basic directions of the regional development, their characteris and analysis of relationships. Also, it is presented a procedure the formation of the integral indicator for the living quality co tions which is based on the usage of both information from expe and accounting correlational interrelations of the basic directi indices. As a result, the integral indicator of the living quality co tions has been estimated for the Russian Federation regions us the data for 2015-2016 years, and it is determined the relat ships between the obtained integral indicator of living quality co tions with the regional development characteristics in the ba directions.

INTRODUCTION

Indices in the basis of differentiation characteristics. The novelty of the proposed approach to the formation of indices for different directions of the regional socio-economic development and the integral indicator of the living quality conditions is determined by the fact that all indices are constructed in the space of regional differentiation characteristics, which are formed and evaluated using theoretically grounded models of regional development. The position of the region in the basis of the differentiation characteristics determines its economic identity. The basis of the regional differentiation characteristics $\mathbf{B}_t^i = \{l_{k,t}, s^1_{k,t}, s^2_{k,t}, te_{k,t}, dte_{k,t})\}_k$ in the time period $[t-1, t]$ consists of five components: $l_{k,t}$ – the scale of the region's i economy at the time t; $te_{k,t}$ – comparable estimator of the technical efficiency; $s^1_{k,t}$ — industry specialization index; $s^2_{k,t}$ — industrialization index; $dte_{k,t}$ — technical efficiency trend, $dte_{k,t} = te_{k,t} - te_{k,t-1}$. As a characteristic of the economy scale, this study uses "economically active population" provided by the Russian Statistical Agency. The basis includes also the first and second principal components of the GRP structure. For the regions of Russian Federation the first principal component separates the regions with the concentration of mining in the structure of GRP from the other regions and could be interpreted as the *index of industry specialization*. The second principal component separates the regions with the concentration of industry (both mining and manufacturing) in the structure of GRP from the other regions (both developed and developing regions) and could be characterized as the *index of industrialization*.

2. FORMATION OF THE INDEX CHARACTERIZING THE DIRECTION OF ECONOMIC DEVELOPMENT.

The index in the chosen direction of social and economic development is based on component analysis in such a way as to be closely related to the essential economic characteristics (basis of the regional differentiation characteristics). Let $I^S(\gamma, \mathbf{y}^k_t) = \sum_i \gamma_{i,t} y^k_{i,t}$ — linear combination of a group of indicators characterizing the direction S of the socio-economic development for the region k, where $\mathbf{y}^k_t = (y^k_{1,t}, \dots, y^k_{N,t})$ — vector of N indicators characterizing the socio-economic direction S for the i-th region at the moment t, $\gamma = (\gamma_1, \dots, \gamma_N)$ – vector of linear combination coefficients for $I^S(\gamma, \mathbf{y}^t_k)$.

Let $IB^S(\delta, \mathbf{B}^k_{t-1}) = \delta_{1,t} l_{k,t-1} + \delta_{2,t} s^1_{k,t-1} + \delta_{3,t} s^2_{k,t-1} + \delta_{4,t} te_{k,t-1} + \delta_{5,t} dte_{k,t-1}$— linear combination of the vector basis components for the i-th region, constructed according to the year $(t-1)$, where $\mathbf{B}^k_t = (l_{i,t}(k), s^1_{i,t}(k), s^2_{i,t}(k), te_{i,t}(k), dte_{i,t}(k))$ and $\delta \in \mathbf{R}^5$. The problem is to determine the values of vector parameters γ, δ, for which $I^S(\gamma, \mathbf{y}_t)$ and $IB^S(\delta, \mathbf{B}_t)$ are most correlated, that is:

$$(\hat{\gamma}, \hat{\delta}) = \underset{\gamma \in \mathbf{R}^N, \delta \in \mathbf{R}^5}{\operatorname{argmax}} \operatorname{corr}(I^S(\gamma, \mathbf{y}_t), IB^S(\delta, \mathbf{B}_{t-1})).$$

The analytical solution to this problem is presented in the articles (Hotelling, 1936; Waugh, 1942; Aivazian, Afanasiev and Kudrov, 2018). The main provisions of the methodology for constructing indices in the basis of the regional differentiation characteristics and the results of its testing in the construction of the index for two directions are presented in the article (Aivazian, Afanasyev and Kudrov, 2019).

3. INDICES FOR THE BASIC DIRECTIONS OF REGIONAL SOCIO-ECONOMIC DEVELOPMENT

Basic directions. At this stage of research, five branches of socio-economic development are considered as determining the life quality. The directions "production of goods and services", "material well-being", "social sphere quality", "population quality" are described and evaluated using the principal component analysis in the monograph (Aivazian, 2012). The direction "social security", for which the relevance of the study is increasing, is described in (Gavrilets et al., 2016). Indicators for the formation of the indices for the basic directions of socio-economic development are presented in the following Table 1.

Table 1. Indicators to form indices

Index	*Indices and indicators*
IB^1	« production of goods and services, volumes», 5 indicators from Russian Statistical Agency[1]: 1 - GRP; 2 - mineral production; 3 - products of manufacturing industries; 4 - agricultural products; 5 - production of electricity, gas, water.
IB^2	« material well-being», 5 indicators from Russian Statistical Agency[2]: 1 - income per capita; 2 - labor productivity index; 3 - migration rate; 4 - unemployment rate; 5 - the share of the population with incomes below the subsistence minimum.
IB^3	« production of goods and services, per capita », 5 indicators from Russian Statistical Agency[3]: 1 - GRP per capita; 2 - mineral production per capita; 3 - products of manufacturing industries per capita; 4 - agricultural products per capita; 5 - production of electricity, gas, water per capita
IB^4	« social sphere quality», 5 indicators from Russian Statistical Agency[4]: 1 - housing construction; 2 - roads length; 3 - fixed capital renewal ratio; 4 - job demand, 5 - employment rate with higher education.
IB^5	« social security», 5 indicators from Russian Statistical Agency[5]: 1 - number of rapes; 2 - the number of thefts; 3 - the number of cases of serious harm to health; 4 - number of murders; 5 – number of robberies (per 100,000 population).
IB^6	« demography», 4 indicators from Russian Statistical Agency[6]: 1 - life expectancy at birth; 2 - total fertility rate; 3 – the share of the population involved in physical education and sports; 4 - population growth.
IB^7	« health», 7 indicators from Russian Statistical Agency[7]: 1 – working-age mortality; 2 - mortality from circulatory diseases; 3 - mortality from oncological diseases (neoplasms); 4 - infant mortality; 5 - respiratory disease mortality; 6 - mortality from diseases of the digestive organs; 7 - mortality from viral hepatitis.
IB^8	« material well-being (subjective) », 5 indicators from Russian Statistical Agency[8]: 1 - share of respondents who able to replace broken furniture; 2 - minimum required monthly income; 3 – share of respondents who can change their family and themselves clothes for the new; 4 - share of respondents need to improve their living conditions; 5 - share of respondents who can buy fruit any time of year.

The group of indicators used in the construction of indices is formed using the methodology of direct links analysis and has an internal structure that reflects their direct connections, which differ from standard correlations. As noted in (Gavrilets, Kudrov and Tarakanova, 2019), «…in the Gaussian case for a collection of m random variables $(X_1, \dots, X_m)$, the absence of the direct connection between X_i and X_j is determined by the equality to zero of the partial correlation coefficient $\rho^{ij} = \rho(X_i, X_j | X_{)i;j(})$, which does not include information $X_{)i;j(} = (X_k | k = 1, \dots, m, k \neq i, j)$. For partial correlation the following equality holds:

$$\rho^{ij} = cor\big(resid(X_i | X_{)i;j(}), resid(X_j | X_{)i;j(}\big),$$

where $resid(X_. | X_{)i;j(})$ - the residuals for the regression of $X_.$ on the variables $X_{)i;j(}$.»

Accordingly, to form the above mentioned eight groups of indicators for each socio-economic direction, there were analyzed the direct links graph. To establish direct connections, the following

[1] Data from Russian Statistical Agency: http://www.gks.ru/bgd/regl/b16_14p/Main.htm
[2] Data from Russian Statistical Agency: http://www.gks.ru/free_doc/doc_2016/rab_sila16.pdf
[3] Data from Russian Statistical Agency: http://www.gks.ru/bgd/regl/b16_14p/Main.htm
[4] Data from Russian Statistical Agency: http://www.gks.ru/free_doc/new_site/inspection/itog_inspect1.htm
[5] Data from Russian Statistical Agency: http://www.gks.ru/free_doc/new_site/inspection/itog_inspect1.htm
[6] Russian Statistical Agency: https://fedstat.ru/
[7] Statistics: Health care in Russia: http://www.gks.ru/bgd/regl/b17_34/Main.htm
[8] Data from Russian Statistical Agency: http://www.gks.ru/free_doc/new_site/inspection/itog_inspect1.htm

hypotheses were checked: $H_0^{ij}:\rho^{ij} = 0$ against $H_1^{ij}:\rho^{ij} \neq 0$ for all possible pairs (i, j) of indicators and significant partial correlations were revealed. The analysis shows that there are fewer direct links than it might seem when analyzing the matrix of pair correlations. Interrelation patterns of differentiation characteristics and indicators, including direct links for indicators IB^1 "production of goods and services, volumes" and IB^2 "material well-being", are given by the authors in the articles (Ayvazyan, Afanasyev, Kudrov, 2018c, 2018d). The indicators used in the formation of eight indices for each correspondent socio-economic direction, shown in Table 1, were selected from a set of 98 initially considered indicators. Seven indices IB^i, $i = 1,...,7$, are based on objective indicators, one IB^8 - on the basis of subjective assessments - the results of sociological surveys. Table 2 shows the coefficients for the components of the basis differentiation characteristics which are used for forming the indices of the basic directions, estimated for the data from the 2015 year and 2016 year. The coefficients under the scale of the economy, technical efficiency and the first two principal components of the GRP structure are stable in time for all indicators.

Table 2. Characteristics of differentiation in the indices for the 2015 – 2016 years

	Coefficients in the indices estimated using data for the 2015 year					*Coefficients in the indices estimated using data for the 2016 year*				
	l	te	s1	s2	dte	l	te	s1	s2	dte
IB^1	0.968	0.051	-0.056	0.048	0.043	0.970	0.049	-0.052	0.054	0.000
IB^2	0.681	0.390	0.275	0.126	0.452	0.625	0.499	0.456	-0.050	0.262
IB^3	0.175	0.207	0.443	0.758	0.085	0.318	0.223	0.457	0.671	0.067
IB^4	0.933	0.301	0.041	-0.164	-0.024	0.928	0.299	-0.049	-0.163	0.052
IB^5	0.604	-0.168	-0.045	0.017	-0.082	0.656	-0.183	-0.038	0.020	-0.140
IB^6	0.092	0.183	0.714	-0.642	-0.134	0.061	0.203	0.847	-0.471	0.138
IB^7	-0.163	0.087	-0.285	-0.485	-0.145	-0.250	0.085	-0.420	-0.759	-0.385
IB^8	0.108	-0.122	-0.468	0.694	0.352	0.037	-0.194	-0.526	0.829	-0.187

Table 3. Correlation matrix for the indices of basic directions estimated for the data from the 2016 year

2016	*IB^1*	*IB^2*	*IB^3*	*IB^4*	*IB^5*	*IB^6*	*IB^7*	*IB^8*
IB^1	1							
IB^2	0.633	1						
IB^3	0.456	0.724	1					
IB^4	0.957	0.745	0.406	1				
IB^5	0.768	0.024	0.058	0.635	1			
IB^6	-0.165	0.514	0.163	0.002	-0.731	1		
IB^7	-0.338	-0.543	-0.89	-0.225	-0.049	-0.058	1	
IB^8	0.302	-0.23	0.308	0.092	0.681	-0.875	-0.363	1

In Table 3 it is shown the correlation matrix of the indices for basic directions, estimated using data from the 2016 year. There is a high positive correlation of the index IB^1 "production of goods and services, volumes" and IB^4 "quality of the social sphere". In these indices, the highly influential component of the basis is the scale of the economy. Index IB^6 "demography" is negatively correlated with IB^8 "material well-being (subjective)". The IB^7 index "health" is negatively correlated with all indicators characterizing the material conditions of life. The highest negative correlation with IB^7 is "production of goods and services, per capita". There is a high negative correlation between indices IB^5 "social security" and IB^6 "demography".

4. FORMATION OF THE INTEGRAL INDICATOR OF LIVING QUALITY CONDITIONS

Aggregation of the indices for basic directions. The approach presented in this section is based on the representation of the integral indicator as a linear combination of the basic directions indices. At the same time, all indices of the basic directions of socio-economic development can be used as independent ones. However, as it was shown in Table 3, some of the eight indices constructed are strongly positively correlated. For example, for data from the 2016 year the correlation coefficient of IB^1 "production of goods and services, volumes" and IB^4 "social sphere quality" is 0.957. Therefore, it is advisable to aggregate some indicators of the basic directions using, for example, the principal component analysis. At the first stage of the aggregation procedure, the first principal component $PC1(IB^1 - IB^8)$ was estimated for the covariance matrix of eight indices $(IB^1, \dots, IB^8)$. The weights for the first principal component are presented in column (3) of Table 4, which are positive except the weights at IB^6 «demography» and IB^7 «health». The growth of the index with the positive weight leads to an increase of the integral indicator.

Negative signs of the weights which correspond to "demography" and "health" do not allow us to consider the first principal component $PC1(IB^1- IB^8)$ as the integral index. At the same time, the signs of weights for the first principal component $PC1(IB^1 - IB^5, IB^8)$, estimated for the covariance matrix of all indices except "demography" and "health", correctly represent the indices $(IB^1 - IB^5, IB^8)$ and then $PC1 (IB^1 - IB^5, IB^8)$ can be used as the integral indicator for the life quality. In this case, we have the set of three indices: the aggregate $PC1(IB^1 - IB^5, IB^8)$ and two initial indices IB^6 "demography" and IB^7 "health". As shown in Table 3, the indices "demography" and "health" are weakly correlated. Therefore their statistical aggregation is impractical. Thus, the set of these three indicators is minimal, taking into account their relationship.

The formation of the integral indicators and the choice of the number of components in them should be carried out accounting the expert's opinions. You can, for example, note (see column (4) of table 4) that IB^8 "material well-being (subjective)" has a low weight in the aggregate $PC1(IB^1 – IB^5, IB^8)$. If it is desirable by experts to increase the significance of objective assessments in the integral indicator, then IB^8 should be used as an independent index, and the statistically aggregated index should be constructed as the first principal component $PC1 (IB^1 – IB^5)$ based on five indices. It is presented in column (5) of Table 4. Note that the share of explained variance by the first principal component $PC1(IB^1 – IB^5)$ is 55.6% and exceeds the share of explained variance by $PC1 (IB^1 – IB^5, IB^8)$ which is 46.8%.

Accordingly, $PC1(IB^1 – IB^5)$ and three initial indices IB^6, IB^7, IB^8 can be used as components for the description of the considered above socio-economic directions. Moreover, $PC1(IB^1 - IB^5)$ can be interpreted as the index of the material basis of life, estimated on objective data.

Table 4. The weight of the first principal components by the indices groups using data from the 2016 year.

	IB	*PC1(IB^1 - IB^8)*	*PC1(IB^1 - IB^5, IB^8)*	*PC1(IB^1 - IB^5)*	*PC1(IB^1, IB^4, IB^5)*	*PC1(IB^2, IB^3)*	*IB^6*	*IB^7*	*IB^8*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(9)	(10)	(8)
	PC%	37.7	46.8	55.6	72.4	86.1	100	100	100
IB^1	*production of goods and services, volumes*	0.493	0.529	0.527	0.623				
IB^2	*material well-being*	0.401	0.439	0.463		0.707			
IB^3	*production of goods and services, per capita*	0.396	0.403	0.409		0.707			
IB^4	*social sphere quality*	0.464	0.510	0.516	0.596				
IB^5	*social security*	0.304	0.293	0.272	0.507				
IB^6	*demography*	-0.115					1		
IB^7	*health*	-0.294						1	
IB^8	*material welfare (subjective)*	0.180	0.135						1

Table 5. Correlation matrix estimated for the data from the 2016 year

Correlation matrix	*PC1(IB^1 - IB^8)*	*PC1(IB^1 - IB^5, IB^8)*	*PC1(IB^1 - IB^5)*	*PC1(IB^1, IB^4,IB^5)*	*PC1(IB^2, IB^3)*	*IB^6*	*IB^7*	*IB^8*
PC1(IB^1 – IB^8)	1							
PC1(IB^1 - IB^5, IB^8)	0,981	1						
PC1(IB^1 - IB^5)	0,969	0.996	1					
PC1(IB^1,IB^4,IB^5)	0,869	0.889	0.874	1				
PC1(IB^2,IB^3)	0,746	0.761	0.783	0.390	1			
IB^6	-0,112	-0.052	0.004	-0.253	0.285	1		
IB^7	-0,552	-0.4716	-0.464	-0.225	-0.625	-0.058	1	
IB^8	0,356	0.269	0.213	0.328	0.044	-0.875	-0.363	1

The aggregate PC1(IB^1 – IB^5), in turn, can be divided into two first principal components PC1(IB^1, IB^4, IB^5) and PC1 (IB^2, IB^3). They are presented respectively in (6) and (7) columns of Table 4. With high explanatory power of these first principal components, their correlation is low (0.390). Therefore, if you wish to differentiate the influence of index IB1 “production of goods and services, volumes” and IB^3 “production of goods and services per capita”, the integral index can be formed on the basis of two aggregates PC1(IB^1, IB^4, IB^5), PC1(IB^2, IB^3) and three indices IB^6, IB^7, IB^8.

Model for the formation of integral indicator. Further, we proceed from the fact that based on principal component analysis for the indices, characterizing the basic directions of socio-economic development, the aggregates necessary for constructing an integral indicator of the life quality are formed. Naturally, the total number of these aggregates and the initial indices is not less than two and does not exceed the number of initial indices characterizing the basic directions. The latter case means that, as a result of principal component analysis, the use of aggregated indices was considered inexpedient.

Background. From the set of aggregated and initial indicators, using for the construction of the integral indicator of life quality, a target indicator can be selected.

Further, the target index is denoted as ICB^0, and other aggregates and indices - ICB^1, ..., ICB^m $(m \geq 1)$. The integral indicator IIB is formed as a linear combination of the indicators $ICB^0, ICB^1, \ldots, ICB^m$ with non-negative coefficients, which in the sum equal to one. The weight vector z * is determined by solving the following optimization problem:

$$z^* = \operatorname{argmax} corr(IIB(z), ICB^0) \quad (1)$$

$$corr(IIB(z), ICB_k) \geq b_k, \quad k = 1, \ldots, m, \quad (2)$$

$$z_0 + z_1 + \ldots + z_m = 1, \quad (3)$$

$$z_0, z_1, \ldots, z_m \geq 0. \quad (4)$$

where $z = (z_0, z_1, \ldots, z_m)$; $IIB(z) = z_0 ICB^0 + z_1 ICB^1 + \ldots + z_m ICB^m$.

In this problem we have $(m + 1)$ variables and $2(m + 1)$ constraints, including conditions for non-negativity of variables. The objective function (1) with $(m + 1)$ variables maximizes the correlation of the integral indicator $IIB(z)$ and the target indicator ICB^0, selected by experts from a set of aggregated indicators and initial indices $ICB^0, ICB^1, \ldots, ICB^m$. The system of m constraints (see 2) describes the correlation relationship between the integral indicator and non-target indicators. With varying parameters b_k, problem (1-4) can be considered as a multi-criteria optimization problem. Moreover, the degree of influence of each from the $(m + 1)$ criteria on the set of Pareto-optimal plans is determined by the expertly given parameters $b_k, k = 1, \ldots, m$. However, we should note that some values have qualitative features. When the value b_k is close to 0.3, the constraint (2) implies a significant positive correlation relationship between the integral indicator and the ICB^k indicator. When the value b_k is close to –0.3, constraint (2) is a condition for the inconsistency of the integral indicator and the ICB^k indicator. If the value of b_k is close to 0, constraint (2) can be considered as a weak condition for consistency. In applied problems, precisely these three types of constraints are of primary interest in the expert formation of the model (1-4). Constraints of the form (2) are not necessarily formed for each non-target indicator. Formally, the absence of such restriction means that the right-hand side in the corresponding inequality is –1.

Problem (1-4) can be represented as a nonlinear optimization problem and solved by numerical methods. Sometimes expert limitations can be imposed not only on the indicators but also on the ranks of individual groups of regions for the given integral indicator. The formalization of such tasks also presents no particular difficulties, but it also requires the use of Boolean variables, which makes it challenging to find a global optimum. As an alternative approach to solving the problem (1-4), an imitation method can be used, which makes it possible to evaluate the necessary correlation coefficients and take into account additional expert recommendations.

Example 1. Integral indicator of life quality. Consider an example of constructing an integral indicator of life quality taking the data for the 2016 year using the aggregate, the first principal component PC1(IB^1 - IB^5), estimated on the basis of five indices for the basic directions (IB^1,..., IB5), as well as indices IB6 "demography" and IB7 "health", formed using objective data. The IB8 indicator “material well-being (subjective)” is used only to assess its relationship with the integral indicator. As a target indicator, we will consider the aggregate PC1(IB1 - IB5) which, as noted above, characterizes the material basis of life. Then $ICB^0 = PC1\,(IB^1 - IB^5)$. Accordingly,

$m - 2, ICB^1 - IB^6, ICB^2 - IB^7$. Considering the positive correlation of PC1(IB1 - IB5) with index IB6, when constraining (2) we restrict ourselves to the conditions of the inconsistency of the integral indicator with IB6 "demography". Since the correlation of PC1(IB1 - IB5) with IB7 indicator is negative, we introduce the condition of a significant positive correlation between the integral indicator and IB7 “health”. In this case, the integral indicator is the solution to the following optimization problem:

$$z^* = \operatorname{argmax} corr(IIB(z), ICB^0)$$

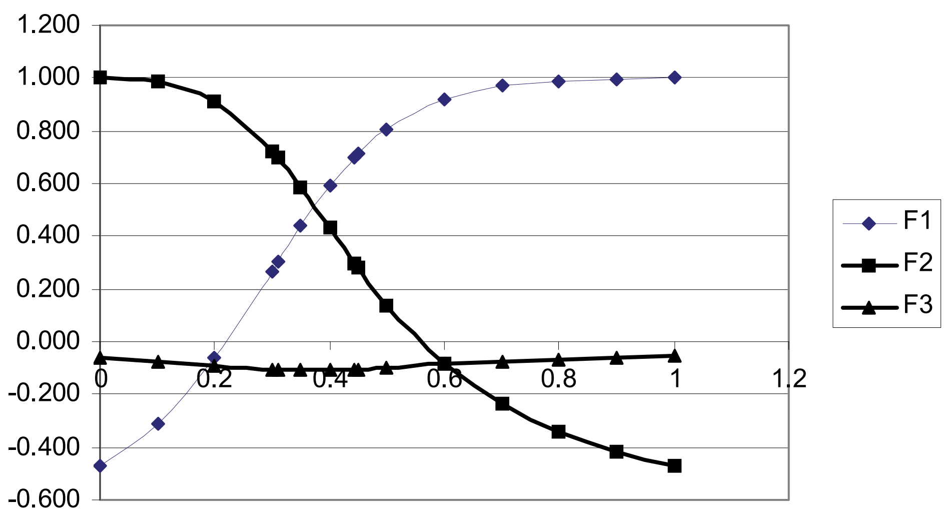
subject to the constraints:

$$corr\,(IIB(z), IB^6) \geq -0.3,$$
$$corr\,(IIB(z), IB^7) \geq 0.3,$$
$$z_0 + z_1 + z_2 = 1,$$
$$z_0, z_1, z_2 \geq 0.$$

where $z = (z_0, z_1, z_3)$; $IIB(z) = z_0 ICB^0 + z_1 IB^6 + z_2 IB^7$.

In Figure 1 on the abscissa axis – the coefficient z0, on the vertical axis - the correlation coefficient $corr(IIB(z), ICB^0)$. The curve F1 describes the increasing dependence of the correlation coefficient of the integral indicator $IIB(z)$ and the target indicator ICB^0 from the weight z_0 taking $z_1 = 0$. The curve F2 describes the decreasing dependence of the correlation coefficient of the integral indicator $IIB(z)$ and IB7 “health” from the weight z_0 taking z_1 = 0. The F3 curve is the dependence of the correlation coefficient of the integral indicator $IIB(z)$ and IB6 “demography” from the weight z_0 taking z_1 = 0. In our example, the peculiarity of the given problem is that for any values of z_0 the restriction $corr(IIB\,(z), IB^6) \geq -0.3$ is fulfilled as a strict inequality. The optimal solution to the considered problem is the vector $z^* - (0.434; 0; 0.566)$, with $corr\,(IIB\,(z^*), ICB^0) = 0.693$, $corr(IIB\,(z^*), IB^6) = -0.043$, $corr\,(IIB\,(z^*), IB^7) = 0.3$, $corr\,(IIB\,(z^*), IB^8) = -0.061$.

Figure 1. Dependence of correlation coefficients from the weight z_0 taking $z_1 = 0$



In column (4) of Table P1 in the appendix there is a list of regions following their ranks by the integral indicator IIB^1. For comparison, in the first column of this table, the regions are ordered according to the ranks of the aggregate $PC1(IB^1 - IB^5)$, which, as noted above, characterizes the material basis of life. Such regions like the Republic of Ingushetia, the Republic of North Ossetia-Alania, the Republic of Dagestan, leading in the rating on IB^7 "health" index (see Table 4), in $PC1(IB^1 - IB^5)$ indicator are in the closing third of the regions. In the integral indicator, which is significantly correlated with IB^7 "health" indicator, these three regions are among the top 10. Such an approach to the forming of integral indicator much changes the idea of life quality, based on the characteristics of its material basis.

Example 2. Integral indicator of life quality. To clarify the role and advantages of the expert approach, instead of the condition $corr(IIB(z), IB^7) \geq 0.3$, which establishes significant positive correlation relationship between the integral indicator and health indicator, we introduce a weak condition of consistency $corr(IIB(z), IB^7) \geq 0$. This condition weakens the role of the "health" indicator IB^7 in the integral indicator as compared with example 1 and strengthens the role of the indicator of the material basis of life $PC1(IB^1 - IB^5)$. In Example 2, the integral indicator is the solution to the following optimization problem:

$$z^* = \operatorname{argmax} corr(IIB(z), ICB^0)$$

subject to the constraints:

$$corr(IIB(z), IB^6) \geq -0.3,$$
$$corr(IIB(z), IB^7) \geq 0,$$
$$z_0 + z_1 + z_2 = 1,$$
$$z_0, z_1, z_2 \geq 0.$$

where $z = (z_0, z_1, z_3)$; $IIB(z) = z_0 ICB^0 + z_1 IB^6 + z_2 IB^7$.

The optimal solution to this problem is the vector $z^* = (0.527; 0; 0.473)$, and we have $corr(IIB(z^*), ICB^0) = 0{,}853$, $corr(IIB(z^*), IB6) = -0{,}03$, $corr(IIB(z^*), IB7) = 0$, $corr(IIB(z^*), IB8) = 0.0013$.

In column (6) of Table P1 (see Appendix) there is a list of regions following their ranks by this integral indicator IIB^2. Compared with example 1, such regions as the Republic of Ingushetia, the Republic of North Ossetia-Alania and the Republic of Dagestan weaken their positions in the integral indicator and occupy the ranks of 10, 17 and 18, respectively. Using data for the 2015 year the integral indicators based on the approach presented here are also calculated. Solutions of optimization problems considered in examples 1 and 2 for the data from the 2015 year lead to the following results:

- The solution to the problem in example 1 for the data from the 2015 year is the vector $z^* = (0.435; 0; 0.565)$. Moreover,

$$corr(IIB(z^*), ICB^0) = 0.677,$$
$$corr(IIB(z^*), IB^6) = 0.088,$$
$$corr(IIB(z^*), IB^7) = 0.3,$$
$$corr(IIB(z^*), IB^8) = -0.079.$$

In the integral indicator such regions like the Republic of Ingushetia, the Republic of North Ossetia-Alania, the Republic of Dagestan are among the 11 best.

- The optimal solution to the problem in example 2 for the data from the 2015 year is the vector $z^* = (0.515; 0; 0.485)$, with

(a) $$corr(IIB\,(z^*), ICB^0) = 0.831,$$

$$corr(IIB\,(z^*), IB6) = 0.024,$$
$$corr\,(IIB\,(z^*), IB7) = 0,$$
(b) $$corr\,(IIB\,(z^*), IB2s) = 0.057.$$

Such regions like the Republic of North Ossetia - Alania, the Republic of Ingushetia and the Republic of Dagestan have ranks 10, 12 and 22, respectively.

In Table 6 it is given the decomposition of the integral indicators, as well as the aggregate PC1(IB^1 - IB^5), in the basis of the regional differentiation characteristics. In the presented integral indicators, the scale of the economy has the highest influence. Technical efficiency is also significant, and in the indicators for the 2016 year - the second principal component of the GRP structure. In PC1(IB^1 - IB^5) for data from the 2015 year all components of the basis are significant; according to the data from the 2016 year, the trend of technical efficiency weakens its importance. Correlation PC1(IB^1 - IB^5) estimated for the data from the 2015 year and the 2016 year is 0.986; correlation of the integral indicators IIB^1 equals to 0.924; correlation of the integral indicators IIB^2 equals to 0.959.

Table 6. Aggregates and integral indicators in the basis.

Year	2016			2015		
Basis	PC1(IB^1-IB^5)	IIB^1	IIB^2	PC1(IB^1-IB^5)	IIB^1	IIB^2
l	1.586	0.564	0.651	1.549	0.596	0.677
te	0.408	0.228	0.243	0.369	0.212	0.226
s1	0.250	-0.123	-0.092	0.214	-0.064	-0.040
s2	0.312	-0.284	-0.234	0.401	-0.092	-0.050
dte	0.102	-0.169	-0.146	0.267	0.038	0.057

In Table 7 it is shown the regressions of integral indicators IIB^1, IIB^2 estimated for the data from the 2016 and 2015 years on the basic directions indices. In these regressions we have relatively high coefficients of determination; all coefficients are significant at the 1% level. In Table 7 it is presented only those directional indices that are significantly interrelated with the integral indicators. In each regression, the influence of health index IB^7 is significant, which corresponds to the condition of the optimization problem (1-4), which was solved for the construction of integral indicators. The influence of PC1(IB^1 - IB^5) is manifested in two ways. The first way (sustainable for two years (2015-2016)) - through IB^2 *“material well-being”* and IB^5 *“social security”*; the second way - through IB^1 *“production of goods and services, volumes”* and IB^3 *“production of goods and services, per capita”*, but the latter is significant only for the data from the 2016 year. Thus, there is a significant correlation of the formed integral indicators of the life quality with five indices: IB^1 “production of goods and services, volumes”, IB^2 “material well-being”, IB^3 “production of goods and services, per capita”, IB^5 “social security”, IB^7 “health”. Explaining the ability of R-squared of most models for the 2016 year is in the range of 0.77 - 0.8.

Table 7. The interrelation of integral indicators IIB^1, IIB^2 and the indices of the basic directions according to the data from the 2016 y. and 2015 y.

	IIB¹ 2016	*IIB² 2016*	*IIB¹ 2016*	*IIB² 2016*	*IIB¹ 2015*	*IIB² 2015*	*IIB¹ 2015*	*IIB² 2015*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Number of regions	80	80	80	80	80	80	80	80
R^2	**0.794**	**0.775**	**0.793**	**0.773**	**0.764**	**0.863**	**0.703**	**0.718**
IB¹ **production of goods and services, volumes**			**0.554** (0.041)	**0.667** (0.049)			**0.551** (0.043)	**0.645** (0.050)
IB² **material well-being**	**0.485** (0.042)	**0.584** (0.051)			**0.446** (0.043)	**0.508** (0.046)		
IB³ **production of goods and services, per capita**			**0.263** (0.084)	**0.316** (0.102)				
IB⁵ **social security**	**0.409** (0.035)	**0.493** (0.043)			**0.394** (0.036)	**0.467** (0.039)		
IB⁷ **health**	**0.490** (0.042)	**0.387** (0.051)	**0.627** (0.035)	**0.552** (0.096)	**0.439** (0.043)	**0.368** (0.046)	**0.363** (0.043)	**0.294** (0.097)

In parentheses are standard errors, all estimates are significant at the 1% level

In Table 8 for the data from the 2016 year, it is shown the regressions of the integral indicators IIB^1, IIB^2 on the indicators of the basic directions for 38 regions with evenly developed sectors in the structure of GRP (Ayvazyan, Afanasyev, Kudrov, 2016b). In these regressions, the significance of IB^1 "production of goods and services, volumes" increases. At the same time, the effect of indicator IB^3 "production of goods and services, per capita" becomes insignificant. There is a significant correlation of the formed integral indicators of life quality with the indicators: IB^1 "production of goods and services, volumes", IB^2 "material well-being", IB^5 "social security", IB^7 "health". The explaining ability R-squared of the considered models for this group of regions is higher than for the entire population and is in the range of 0.83 - 0.87.

Table 8. The interrelation of integral indicators IIB^1, IIB^2 and indices of the basic directions for the data from the 2016 year considered for 38 regions with evenly developed sectors in GRP.

	IIB¹ 2016	*IIB² 2016*	*IIB¹ 2016*	*IIB² 2016*
(1)	(2)	(3)	(4)	(5)
Number of regions	38	38	38	38
R^2	**0.863**	**0.865**	**0.836**	**0.834**
IB¹ **production of goods and services, volumes**			**0.632** (0.062)	**0.759** (0.075)
IB² **material well-being**	**0.514** (.060)	**0.618** (.072)		
IB³ **production of goods and services, per capita**			*0.157* *(0.170)*	*0.189* *(0.204)*
IB⁵ **social security**	**0.480** (.056)	**0.577** (.068)		
IB⁷ **health**	**0.625** (0.084)	**0.549** (0.101)	**0.629** (0.154)	**0.554** (0.185)

All estimates, except those shown in italics, are significant at the 1% level.

In Table 9 it is shown the weighted mean aggregated indices and ranks of federal districts in terms of life quality. When weighted aggregating the indices of the regions within the federal district, we took the weights equal to the share of the population in the correspondent region in the total population of the federal district. The column (3) shows the ranks of federal districts in weighted aggregates of PC1(IB^1 - IB^5) for the 2016 year. In columns (4) and (5) it is presented the weighted aggregates of PC1(IB^1 - IB^5) for 2015 and 2016 years. In column (6) are ranks of federal districts by aggregated with weights integral indicator IIB^2. In columns (7) and (8) - aggregated with weights integral indicator IIB^2 for 2015 and 2016 years. In column (2) is the difference in ranks given in columns (3) and (6), which shows on how many positions the rank of the federal district by the integral indicator increased in comparison with the rank given by the indicator PC1(IB^1 - IB^5).

Table 9. Indicators and ranks of Federal Districts in terms of life quality.

		aggregate PC1(IB^1 - IB^5)			*integral indicator IIB^2*		
Federal District	*(3)-(6)*	*rank PC1*	*indicator 2015*	*indicator 2016*	*rank IIB^2*	*indicator 2015*	*indicator 2016*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Central	0	1	3.689	3.664	1	1.760	1.935
Northwestern	0	2	2.179	2.168	2	0.864	0.933
Southern	1	4	0.814	0.792	3	0.521	0.614
North Caucasus	3	8	-0.922	-0.865	5	0.170	0.131
Volga	-1	5	0.709	0.471	6	0.115	-0.015
Ural	-1	3	1.670	1.735	4	0.246	0.397
Siberian	-2	6	-0.443	-0.357	8	-0.411	-0.451
Far Eastern	0	7	-0.525	-0.372	7	-0.228	-0.184

The central and northwestern federal retain, respectively, the first and second positions both for integral indicator IIB^2, formed taking into account "health" IB^7, and for indicator PC1(IB^1 - IB^5), which characterizes the material basis of life. According to the integral indicator, the rank of the Far Eastern Federal District also coincides with the rank of the PC1(IB^1 - IB^5). These three federal districts are resistant to the transition from PC1(IB^1 - IB^5) to integral indicator IIB^2. The ranks of the Southern, Volga and Ural Federal Districts changed by one. These federal districts can be considered as weakly stable. The rank of the Siberian Federal District has deteriorated in two positions. The rank of the North Caucasus Federal District improved by three positions. These two federal districts can be characterized as unstable when moving from the indicator PC1 (IB^1 - IB^5), which reflects the material basis of life to the integral indicator IIB^2, which takes "health" into account. Thus, the ranks of six from the eight considered Federal Districts are stable or weakly stable for two ways of assessment: by PC1(IB^1 - IB^5) estimates and by the integral indicator IIB^2. The ranks of the two federal districts - Siberian and North Caucasus are not stable.

CONCLUSION

It has been formed a basis which consists of five differentiation characteristics obtained based on theoretically grounded models of regional development. In this basis of the differentiation characteristics, eight indicators have been constructed, characterizing the five basic directions of socio-economic development of the Russian Federation regions: production of goods and services, material well-being, population quality, social sphere quality, internal security. The indices constructed in the basis are correlated as much as possible with the index formed using the group of indicators characterizing the correspondent socio-economic direction.

The indicators characterizing the material basis of life are constructed based on a group of indicators selected as a result of the graph of direct links analysis, constructed using the coefficients of partial correlations. Indicators of the directions "demography" and "health" are formed on the basis of regulatory materials.

An aggregate of five indicators was constructed, which characterizes the material basis of life, which, together with the indicators of demography and health, can be used to construct an integral indicator of the living quality conditions. It is presented a model for the formation of an integral indicator and the results of its approbation according to the data for 2015 and 2016 years.

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Appendix

Table P1. Ratings for PC1(IB1 -IB5) and Integrated Indicators for the 2016 year

	PC1(IB¹-IB⁵)		*IIB^1= 0.434 PC1(IB^1 - IB^5) + 0.566 IB^7*		*IIB^2= 0.527 PC1(IB^1 - IB^5) + 0.473 IB^7*
(1)	(2)	(3)	(4)	(5)	(6)
1	Moscow	1	Moscow	1	Moscow
2	St. Petersburg	2	St. Petersburg	2	St. Petersburg
3	Moscow region	3	Moscow region	3	Moscow region
4	Tyumen region	4	Krasnodar region	4	Krasnodar region
5	Republic of Tatarstan	5	Voronezh region	5	Voronezh region
6	Sverdlovsk region	6	Kamchatka Krai	6	Tyumen region
7	Krasnodar region	7	The Republic of Ingushetia	7	Kamchatka Krai
8	Sakhalin region	8	Belgorod region	8	Belgorod region
9	Leningrad region	9	Republic of North Ossetia - Alania	9	Rostov region
10	Nizhny Novgorod Region	10	The Republic of Dagestan	10	The Republic of Ingushetia
11	Belgorod region	11	Rostov region	11	Republic of Tatarstan
12	Chukotka Autonomous Region	12	Chechen Republic	12	Sverdlovsk region
13	Rostov region	13	Bryansk region	13	Chukotka Autonomous Region
14	Voronezh region	14	Primorsky Krai	14	Leningrad region
15	Republic of Bashkortostan	15	Stavropol region	15	Republic of Bashkortostan
16	Krasnoyarsk region	16	Chukotka Autonomous Region	16	Kaliningrad region
17	Samara Region	17	Tyumen region	17	Republic of North Ossetia - Alania
18	Kaliningrad region	18	Oryol Region	18	The Republic of Dagestan
19	Kaluga region	19	Republic of Bashkortostan	19	Bryansk region
20	Lipetsk region	20	Kaliningrad region	20	Primorsky Krai
21	Chelyabinsk region	21	Kabardino-Balkaria	21	Stavropol region
22	Perm region	22	Leningrad region	22	Chechen Republic
23	Tula region	23	Sverdlovsk region	23	Nizhny Novgorod Region
24	Omsk region	24	Republic of Tatarstan	24	Lipetsk region
25	Novosibirsk region	25	Kursk region	25	Kursk region
26	Yaroslavl region	26	Republic of Adygea	26	Oryol Region
27	Kursk region	27	Mari El Republic	27	Kaluga region
28	The Republic of Sakha (Yakutia)	28	Lipetsk region	28	Kabardino-Balkaria
29	Magadan Region	29	Tver region	29	Republic of Adygea
30	Kamchatka Krai	30	Altai Republic	30	Yaroslavl region
31	Vologodskaya Oblast	31	Novgorod region	31	Samara Region
32	Stavropol region	32	Amur region	32	Novosibirsk region
33	Khabarovsk region	33	Yaroslavl region	33	Sakhalin region
34	Vladimir region	34	Saratov region	34	Novgorod region

35	Novgorod region	35	Novosibirsk region	35	Tambov Region
36	Bryansk region	36	Nizhny Novgorod Region	36	Tver region
37	Murmansk region	37	Kaluga region	37	Omsk region
38	Arkhangelsk region	38	The Republic of Buryatia	38	Saratov region
39	Primorsky Krai	39	Astrakhan region	39	Khabarovsk region
40	Republic of Adygea	40	Tambov Region	40	Mari El Republic
41	Saratov region	41	Smolensk region	41	Smolensk region
42	Komi Republic	42	Pskov region	42	Volgograd region
43	Irkutsk region	43	Volgograd region	43	Chelyabinsk region
44	Smolensk region	44	Khabarovsk region	44	Amur region
45	Ulyanovsk region	45	The Republic of Mordovia	45	Astrakhan region
46	Volgograd region	46	Kostroma region	46	The Republic of Buryatia
47	Orenburg region	47	Karachay-Cherkess Republic	47	Pskov region
48	Ryazan Oblast	48	Ivanovo region	48	The Republic of Mordovia
49	Udmurtia	49	Omsk region	49	Ivanovo region
50	Ivanovo region	50	Samara Region	50	Ryazan Oblast
51	Kemerovo region	51	Ryazan Oblast	51	Tula region
52	Tver region	52	Republic of Kalmykia	52	Kostroma region
53	Oryol Region	53	Jewish Autonomous Region	53	Krasnoyarsk region
54	Tomsk region	54	Tyva Republic	54	Altai Republic
55	Tambov Region	55	Chelyabinsk region	55	Karachay-Cherkess Republic
56	The Republic of Mordovia	56	Altai region	56	Magadan Region
57	Astrakhan region	57	Sakhalin region	57	Vladimir region
58	Kabardino-Balkaria	58	Republic of Karelia	58	Vologodskaya Oblast
59	Altai region	59	Transbaikal region	59	Perm region
60	Pskov region	60	Tula region	60	Altai region
61	The Republic of Dagestan	61	Vladimir region	61	Ulyanovsk region
62	The Republic of Buryatia	62	Penza region	62	Republic of Kalmykia
63	Amur region	63	Magadan Region	63	Jewish Autonomous Region
64	Republic of North Ossetia - Alania	64	Ulyanovsk region	64	Republic of Karelia
65	Kurgan region	65	Vologodskaya Oblast	65	Murmansk region
66	Chuvash Republic	66	Chuvash Republic	66	Penza region
67	Mari El Republic	67	Krasnoyarsk region	67	Irkutsk region
68	Penza region	68	Kurgan region	68	Transbaikal region
69	Kostroma region	69	Irkutsk region	69	Arkhangelsk region
70	Chechen Republic	70	Murmansk region	70	Chuvash Republic
71	Kirov region	71	Perm region	71	Tyva Republic
72	Republic of Karelia	72	Kirov region	72	Kurgan region
73	Karachay-Cherkess Republic	73	Arkhangelsk region	73	Orenburg region
74	Transbaikal region	74	Orenburg region	74	Kirov region
75	The Republic of Khakassia	75	Tomsk region	75	Tomsk region
76	The Republic of Ingushetia	76	Udmurtia	76	The Republic of Sakha (Yakutia)
77	Altai Republic	77	The Republic of Khakassia	77	Udmurtia
78	Republic of Kalmykia	78	Komi Republic	78	Komi Republic
79	Jewish Autonomous Region	79	Kemerovo region	79	Kemerovo region
80	Tyva Republic	80	The Republic of Sakha (Yakutia)	80	The Republic of Khakassia