PROPOSITION OF MATHEMATICAL MODEL IN THE FUNCTION OF OPTIMIZATION OF THE TRAFFIC SYSTEM IN SISAK-MOSLAVINA COUNTY

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Summary
The main purpose of this paper is to introduce traffic system of Sisak-Moslavina County using a mathematical model. That approach demands that the relevant elements of the model are defined, as well as scientifically based determination of optimal or sufficient values of variables of the traffic system. To quantify optimal values of variables growth matrix is used, which treats structural relations of elements of the model in traffic system in a specific way. That particularity is seen in the fact that elements of the model in traffic system are mutually dependent, and that the tracking of their value in particular time periods should be observed simultaneously through direct and indirect growth rates. The research hypothesis is that by set the model in the function of the optimization of the traffic system we can induce economic development of Sisak-Moslavina County.
Key words*: mathematical model, growth matrix, optimization, traffic system of Sisak-Moslavina County, growth rates*

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1. Introduction

Traffic has a crucial role in the economic activity. Furthermore, it is a primary economic activity which contributes to manufacturing of national product and which is important in its realization (Quinet & Vickerman, 2008,13). Besides direct influences, there are also proven potential positive indirect influences of traffic on the economy (external economy) (Rodrigue J-P et al, 2009). Knowing that, the thesis is that the optimization of the traffic system of Sisak-Moslavina County has great possibilities for development of every branch of traffic, intermodal transport and logistical centers (Zuko, 2011,184).

Considering that, the task of this scientific discussion is to confirm and investigate relevant elements of the model of traffic system of Sisak-Moslavina County, in other words confirmation of scientific hypothesis, which is:

By determining relevant elements of the model and by their quantification, it is possible to determine the trajectory of their development and to optimize traffic system in Sisak-Moslavina County.

The aim of the research is closely related with the set task of research and scientific hypothesis: to present traffic system of Sisak-Moslavina County by using a mathematical model, to determine relevant elements of the model of traffic system in Sisak-Moslavina County, to value elements of the model of traffic system in Sisak-Moslavina County, to calculate direct and indirect growth rates of relevant elements of the model of traffic system in Sisak-Moslavina County, and to optimize development of traffic system in Sisak-Moslavina County.

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determine relevant elements of the model of traffic system in Sisak-Moslavina County, to value elements of the model of traffic system in Sisak-Moslavina County, to calculate direct and indirect growth rates of relevant elements of the model of traffic system in Sisak-Moslavina County, and to optimize development of traffic system in Sisak-Moslavina County.

2. Theory frame of research

Scientific based assumption is that the traffic system is made of n mutually dependent elements. With y_{it} and y_{it-1} will mark the value (as an input) of i element of the model (Heizer & Render, 2010,237) of traffic system of Sisak-Moslavina County, (i=1,..., n) in the period t and t-1.

Growth value of input *i* element of the model of traffic system in Sisak-Moslavina County is

$$\Delta y_{it} = \Delta y_{it} - \Delta y_{it-1} \tag{1}$$

Indirect growth rate of i element of the model of traffic system of Sisak-Moslavina County, in relation to *j* element, is defined as relation of growth input of i-element of the model of traffic system of Sisak-Moslavina County, Δy_{it}, and value of input j-element of the model of traffic system in Sisak-Moslavina County, in the period t, respectively

$$r_{ijt} = y_{it} / y_{jt} \qquad i,j = 1, \ldots., n.\ Y_{jt} \neq 0 \tag{2}$$

Indirect growth rates can be expressed in the form of growth matrix value element of the model of traffic system in Sisak-Moslavina County (Stojanović, 1999,487):

$$Rt = \begin{bmatrix} r_{11} & r_{12} & \ldots & 1_{nt} \\ r_{21} & r_{22} & \ldots & 2_{nt} \\ \ldots & \ldots & \ldots & \ldots \\ r_{n1t} & r_{n2t} & \ldots & r_{nnt} \end{bmatrix} \qquad t = 1, \ldots., T \tag{3}$$

where elements on the main vertical mark direct (i = j), and the rest (i ≠ j) mark indirect growth rates. Elements in i line mark growth value in i element the model of optimization of traffic system in Sisak-Moslavina County in the relation to the value of other elements of the model. Elements in i column mark growth value input in all elements of the model in the relation to input of i element in the period of t.

Based on the preceding it can be determined that every element in the growth matrix is presented with one line and one column, with elements which express indirect or relative growth relations. So, for example, in the first line growth input of the first element the model of traffic system of Sisak-Moslavina County is expressed in relation to the other elements, and in the first column, growth of other elements in the relation on the input of the first element is expressed. Other lines and columns match other growth elements of the model.

Indirect growth rates can also be defined in the relation to values of j element the model in the period t-1, respectively

$$r'_{ijt} = \frac{\Delta y_{it}}{\Delta y_{j,t-1}} \qquad i,j = 1, \ldots., n \tag{4}$$

Relation between indirect growth rates (2) and (4) can be made through following mutual relation:

$$r_{ijt} = r'_{ijt} / 1 + r'_{jjt} \quad \text{and} \quad r_{ijt} = r'_{ijt} / 1 - r'_{jjt} \quad i,j = 1,\ldots, n \qquad (5)$$

Growth matrix can also be determined through external vector of developing elements of the model (Zelenika & Pupavac, 2008,554). That way of determining is useful for practical calculation of growth matrix. Growth vector elements of the model for optimization of traffic system of Sisak-Moslavina County:

$$\Delta y_{it} = \Delta_{lt},\ldots, \Delta y_{mt} \qquad (6)$$

and the vector of reciprocate values of the model elements optimization of traffic system of Sisak-Moslavina County;

$$(1/y_t) = (1/y_{1t}, \ldots, 1/y_{nt}) \qquad y_{it} \neq 0, \quad i,j = 1, \ldots, n. \qquad (7)$$

External value of vector growth the model elements of optimization of traffic system in Sisak-Moslavina County and the vector of reciprocate values define the growth matrix of optimization of traffic system in Sisak-Moslavina County (Stojanović, 1990,489).

$$R_{pt} = \Delta y'_t(1/y_t) = \begin{bmatrix} \Delta y_{1t} \\ \ldots \\ \Delta y_{mt} \end{bmatrix} (1/y_{1t}, \ldots, 1/\Delta y_{nt}) \qquad (8)$$

$$Rpt = \begin{bmatrix} \Delta y_{1t} / y_{1t} & \ldots & \Delta y_{1t} / y_{nt} \\ \ldots & \ldots & \ldots \\ \Delta y_{mt} / y_{1t} & \ldots & \Delta y_{mt} / y_{nt} \end{bmatrix} = \begin{bmatrix} r_{11t} & \ldots & r_{1nt} \\ \ldots & \ldots & \ldots \\ r_{m1t} & \ldots & r_{mnt} \end{bmatrix} \qquad (9)$$

When the direct growth rates are observed, then the growth of one element is expressed independently from the growth of others. However, when indirect growth rates are defined, more accurately growth of *i* element in the relation on *j* (i,j = 1, ...,n), it is possible to determine the structure of growth elements and to express all relations through growth matrix in the overall system. By simultaneously expressing direct and indirect growth percentage it is possible to track changes in intensity of growth elements and their structural relations.

3. Materials and the methods of research

To determine relevant elements of the model, besides theoretical research, field research was done. Field research was made through the method of poll and in two steps (Kotler & Armstrong, 2009,115). First step had begun with 35 independent elements which influence growth and development of traffic system in Sisak-Moslavina County. The preposition of factors which influence growth and development of traffic system in Sisak-Moslavina County through poll questionnaire and through Internet was forwarded to traffic, logistic and manufacturing businesses managers in Sisak-Moslavina County. Their task was to point out ten relevant elements of the model, or to add some of the elements which, in their opinion, were not in the preposition. Based on the research (answers were gathered and processed from 143 managers from all levels), there were 19 independent elements of the model pointed out, plus other elements (population, macro transformation of traffic control) as twentieth element of the model. In the second step those 20 pointed out relevant elements of the model were forwarded for valuation to only traffic and logistic businesses managers in Sisak-Moslavina County. Their task was to determine the beginning values of relevant elements of the model for the year 2008 (taking into the account their status and meaning in narrow and wide gravitation area) and their expected values in the year 2015 when the first positive effects of membership of Republic of Croatia in the European Union are expected, and also the approach of other countries of southeastern Europe to full

membership in EU, and in the year 2025 when it is expected that all the countries of southeastern Europe will be full members of EU and together coordinate development of regional traffic system. Evaluating was made on the scale from 1 to 100, and the results of the evaluation of 58 managers are in the table 1.

Table 1: Value of relevant elements of the model in the function of optimization of traffic system of Sisak-Moslavina County

No	*Developmental elements*	*Input* Y_{it}			*Growth*
		2008	*2015*	*2025*	$\Delta y_{i,2025}$
1.	Traffic infrastructure	40	65	90	50
2.	Traffic superstructure	30	55	80	50
3.	Human potentials in traffic	50	70	90	40
4.	Commodity flows	30	65	90	60
5.	Travelling flows	30	55	80	50
6.	Transport technologies	20	45	75	55
7.	Information technologies	30	65	90	60
8.	Financial potentials	30	50	85	55
9.	Traffic ecology	35	65	90	55
10.	Transport chains and transport grids	30	55	80	50
11.	Educational systems	50	70	90	40
12.	Energetic efficiency of traffic	40	60	85	45
13.	Gross domestic product	40	65	90	50
14.	Traffic politics	50	65	85	35
15.	Number and structure of traffic businesses	40	65	80	40
16.	Number and structure of logistic businesses	25	50	70	45
17.	Number and structure of manufacturing businesses	50	60	75	25
18.	Traffic safety	30	55	85	55
19.	Urban and space plans	55	70	90	35
20.	Other elements of the model	50	70	80	30

Source: authors

4. Results and discussion

Based on the information from table 1, growth matrix was determined by elements of the model of optimization of traffic system in Sisak-Moslavina County, and more developed regional European traffic system in relation to current and future values in the period 2008/2025 (table 2).

In table 2 direct and indirect growth rates for every relevant element of the model of optimization of traffic system in Sisak-Moslavina County are clearly shown. Taking into account the direct growth rates, it can be established that transport technologies (73,33%), traffic safety (68,75%), commodity flows (66,67%), number and structure of logistic businesses (64,29%) and financial potentials (64,71%) will be decisive elements for optimization of traffic system in Sisak-Moslavina County. These results point out the need to realize qualitative goals in the function of optimization for traffic system in Sisak-Moslavina County, more to the point:

a) to achieve appropriate valuation of advantages of geotraffical position of Sisak-Moslavina County across national and regional traffic grid, b) qualitatively interconnect counties and cities in Sisak-Moslavina County with connections to traffic corridors, corridor X and corridor Vc, c) to establish more dynamic economic growth by developing traffic infrastructure and traffic industry, d) to reduce all types of traffic expenses and by doing that act on improving general prosperity levels and e) building the town of Sisak as a traffic junction of the county. Research has shown that smaller and also very high growth rates will have the following elements of the model: traffic in-

frastructure (55,56%), traffic substructure (62,50%), travelling flows (62,50%), information technologies (55,56%), traffic ecology (61,11%) and transport chains and transport grids (62,50%).

Relatively high growth rates of these elements of the model show the necessity of maintaining already existing and building new infrastructural contents, so the area of Sisak-Moslavina County wouldn't be passed by, which will cause shutting out a great part of the County from the possibility of economic recovery, and then faster economic growth. Namely, flow of traffic only through corridor roads does not ensure an even growth of traffic system of the County, and does not use its full potentials.

Based on the information from table 2 indirect growth rates between individual relevant elements of the model of optimization for traffic system in Sisak-Moslavina County can be foreseen. In continuation of this paper rest of relevant elements the model of optimization for traffic system in Sisak-Moslavina County will be compared with: traffic flows, transport technologies and traffic safety as elements of the model with highest direct growth rates. Comparison of other elements is possible with the same principle, but for the sake of rationalization is left out.

By comparing commodity flows with other elements of the model (table 2, line 4) which influence the optimization of traffic system in Sisak-Moslavina County for period 2008/2025, a boost in growth rates of commodity flows of 85,71% in the relation to the number and structure of manufacturing businesses, 75% in relation to transport chains and transport grids, 75% in relation to number and structure of traffic businesses, and so on can be seen. So, high growth rates of commodity flows in relation to named elements show strong interconnection of these elements of the model is, ie. the fact that these elements the model are under strong influence of commodity flows. For example, growth of commodity flows in traffic system in Sisak-Moslavina County will result in increase of number and size of manufacturing, logistic and traffic businesses, maintaining transport chains and developing transport grids which will contribute to further growth of commodity flows.

By comparing transport technologies with other elements of the model (table 2, line 4) which influence the optimization of traffic system in Sisak-Moslavina County for period 2008/2025, a boost in growth rates of transport technologies of 78,57% in relation to number and structure of logistic businesses, 73,33% in relation to number and structure of manufacturing businesses, 68,75% in relation to number and structure of traffic businesses, 68,75% in relation to traffic safety, and so on, can be seen. So, high growth rates of transport technologies in relation to named elements point to the fact that these elements of the model are under strong influence of use of transport technologies. For example, a boost in use of transport technologies in traffic system in Sisak-Moslavina County will result in a boost in number of manufacturing, logistic and traffic businesses, and more importantly in a boost in traffic safety.

By comparing traffic safety with other elements of the model (table 2, line 18) which influence the optimization of traffic system in Sisak-Moslavina County for period 2008/2025, a boost in growth rates of traffic safety of 78,57% in relation to number and structure of logistic businesses, 73,33% in relation to number and structure of manufacturing businesses and transport technologies, 68,75% in relation to number and structure of traffic businesses, transport chains and transport grids, travelling flows and other elements of the model can be seen. So, high growth rates in traffic safety in relation to named elements point to the fact that these elements of the model are under strong influence of traffic safety. For example, a boost in safety of traffic system will result in boost in traffic, but also in the need to regulate traffic (other elements of the model).

Table 2: Growth rates of elements of the model of optimization from traffic system in Sisak-Moslavina County for the period 2008-2025

EM	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
1.	55,56%	62,50%	55,56%	55,56%	62,50%	66,67%	55,56%	58,82%	55,56%	62,50%	55,56%	58,82%	55,56%	58,82%	62,50%	71,43%	66,67%	62,50%	55,56%	62,50%
2.	55,56%	62,50%	55,56%	55,56%	62,50%	66,67%	55,56%	58,82%	55,56%	62,50%	55,56%	58,82%	55,56%	58,82%	62,50%	71,43%	66,67%	62,50%	55,56%	62,50%
3.	44,44%	50,00%	44,44%	44,44%	50,00%	53,33%	44,44%	47,06%	44,44%	50,00%	44,44%	47,06%	44,44%	47,06%	50,00%	57,14%	53,33%	50,00%	44,44%	50,00%
4.	66,67%	75,00%	66,67%	66,67%	75,00%	80,00%	66,67%	70,59%	66,67%	75,00%	66,67%	70,59%	66,67%	70,59%	75,00%	85,71%	80,00%	75,00%	66,67%	75,00%
5.	55,56%	62,50%	55,56%	55,56%	62,50%	66,67%	55,56%	58,82%	55,56%	62,50%	55,56%	58,82%	55,56%	58,82%	62,50%	71,43%	66,67%	62,50%	55,56%	62,50%
6.	61,11%	68,75%	61,11%	61,11%	68,75%	73,33%	61,11%	64,71%	61,11%	68,75%	61,11%	64,71%	61,11%	64,71%	68,75%	78,57%	73,33%	68,75%	61,11%	68,75%
7.	55,56%	62,50%	55,56%	55,56%	62,50%	66,67%	55,56%	58,82%	55,56%	62,50%	55,56%	58,82%	55,56%	58,82%	62,50%	71,43%	66,67%	62,50%	55,56%	62,50%
8.	61,11%	68,75%	61,11%	61,11%	68,75%	73,33%	61,11%	64,71%	61,11%	68,75%	61,11%	64,71%	61,11%	64,71%	68,75%	78,57%	73,33%	68,75%	61,11%	68,75%
9.	61,11%	68,75%	61,11%	61,11%	68,75%	73,33%	61,11%	64,71%	61,11%	68,75%	61,11%	64,71%	61,11%	64,71%	68,75%	78,57%	73,33%	68,75%	61,11%	68,75%
10.	55,56%	62,50%	55,56%	55,56%	62,50%	66,67%	55,56%	58,82%	55,56%	62,50%	55,56%	58,82%	55,56%	58,82%	62,50%	71,43%	66,67%	62,50%	55,56%	62,50%
11.	44,44%	50,00%	44,44%	44,44%	50,00%	53,33%	44,44%	47,06%	44,44%	50,00%	44,44%	47,06%	44,44%	47,06%	50,00%	57,14%	53,33%	50,00%	44,44%	50,00%
12.	50,00%	56,25%	50,00%	50,00%	56,25%	60,00%	50,00%	52,94%	50,00%	56,25%	50,00%	52,94%	50,00%	52,94%	56,25%	64,29%	60,00%	56,25%	50,00%	56,25%
13.	55,56%	62,50%	55,56%	55,56%	62,50%	66,67%	55,56%	58,82%	55,56%	62,50%	55,56%	58,82%	55,56%	58,82%	62,50%	71,43%	66,67%	62,50%	55,56%	62,50%
14.	38,89%	43,75%	38,89%	38,89%	43,75%	46,67%	38,89%	41,18%	38,89%	43,75%	38,89%	41,18%	38,89%	41,18%	43,75%	50,00%	46,67%	43,75%	38,89%	43,75%
15.	44,44%	50,00%	44,44%	44,44%	50,00%	53,33%	44,44%	47,06%	44,44%	50,00%	44,44%	47,06%	44,44%	47,06%	50,00%	57,14%	53,33%	50,00%	44,44%	50,00%
16.	50,00%	56,25%	50,00%	50,00%	56,25%	60,00%	50,00%	52,94%	50,00%	56,25%	50,00%	52,94%	50,00%	52,94%	56,25%	64,29%	60,00%	56,25%	50,00%	56,25%
17.	27,78%	31,25%	27,78%	27,78%	31,25%	33,33%	27,78%	29,41%	27,78%	31,25%	27,78%	29,41%	27,78%	29,41%	31,25%	35,71%	33,33%	29,41%	27,78%	31,25%
18.	61,11%	68,75%	61,11%	61,11%	68,75%	73,33%	61,11%	64,71%	61,11%	68,75%	61,11%	64,71%	61,11%	64,71%	68,75%	78,57%	73,33%	68,75%	61,11%	68,75%
19.	38,89%	43,75%	38,89%	38,89%	43,75%	46,67%	38,89%	41,18%	38,89%	43,75%	38,89%	41,18%	38,89%	41,18%	43,75%	50,00%	46,67%	43,75%	38,89%	43,75%
20.	33,33%	37,50%	33,33%	33,33%	37,50%	40,00%	33,33%	35,29%	33,33%	37,50%	33,33%	35,29%	33,33%	35,29%	37,50%	42,86%	40,00%	37,50%	33,33%	37,50%

5. Conclusion

Mathematical model in the function of optimization of traffic system in Sisak-Moslavina County and the analysis of direct and indirect growth rates of relevant elements of the model show the tendency of development and the role of individual elements in the function of optimization of traffic system in Sisak-Moslavina County. It is not possible to say that a particular element is irrelevant in optimization of traffic system in Sisak-Moslavina County. Each of them contributes and has an impact qualitatively as well as quantitatively to the function and development of optimal traffic system in Sisak-Moslavina County, as well as in its involvement in national, regional and European traffic system. However, considering direct and indirect growth rates it can be seen that the transport technologies, traffic safety, commodity flows, number and structure of logistic businesses and financial potentials will be determining factors of optimization of traffic system in Sisak-Moslavina County. This is very important to have in mind because of the global crisis and reduced funds for investments in traffic infrastructure. This data also show the need to transform traffic structure in Sisak-Moslavina County making classic traffic forms insufficient. The main idea is that on these elements the traffic system in Sisak-Moslavina County is created, using natural features such as geographical position, availability of natural, human and material resources, history and cultural growth, and to achieve a degree of industrial development and the size of narrower and wider gravitational area. Given results show the qualitative dimension of leading the traffic strategy, pointing the need for optimal use of existing traffic grid, more efficient organization of managing the traffic grid, evaluation of effects of traffic development on economy, environment and traffic safety, problems of optimal proportion of growth of particular branches of traffic, and traffic system of the County as a whole. High direct growth rates of transport technologies and the number and structure of logistic businesses point to the fact that in the traffic system in Sisak-Moslavina County there is no appropriate coordination or cooperation, which disables the use of contemporary ways of organization of traffic and introduction of intermodal transport. That is one of the limiting factors of involving Croatian traffic system into European traffic grid.

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