SUPPLY CHAIN EQUILIBRIUMS UNDER NON-LINEAR COST FUNCTIONS OF PARTICIPANTS

RAVNOTEŽNO STANJE LANCA SNABDIJEVANJA U USLOVIMA NELINEARNE FUNKCIJE TROŠKOVA UČESNIKA

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***Abstract**: the developed econometric model with non-linear cost functions allows to optimize the economic parameters of the supply chain participants in case of their economic independence. The discovered analytical relations make it possible to determine equilibrium values for tariff, product price and traffic volume, to maximize profit of each supply-chain participant with various transportation tariffs to be charged.*
Key words: tariff, cost function, carrier, balance, leader, exporter.

***Apstrakt**: Razvijen ekonometrijski model sa nelinearnim funkcijama troškova omogućuje optimizaciju parametara lanca snabdijevanja u uslovima ekonomske nezavisnosti učesnika. Otkrivene analitičke zavisnosti omogućuju određivanje ravnoteže tarifa, proizvodne cijene i obima transporta, u cilju maksimiziranja profita za svakog od učesnika u lancu snabdijevanja, za koje se, inače, vezuju različite tarife.*
***Ključne riječi**: tarifa, funkcija troškova, prevoznik, vodeći izvoznik.*

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According to the classical microeconomic theory [1], the delivery quantity optimization of the production to the market is accomplished by means of comparing the producer's income function $D(Q)$ from production sales amount Q and cost function $V(Q)$ for the manufacture and transportation of this volume.

If the manufacturer is supposed to deliver his production to the market by himself (purchase of goods is accomplished on basic terms of delivery DDU), then he becomes a monopolist, i.e. the only seller.

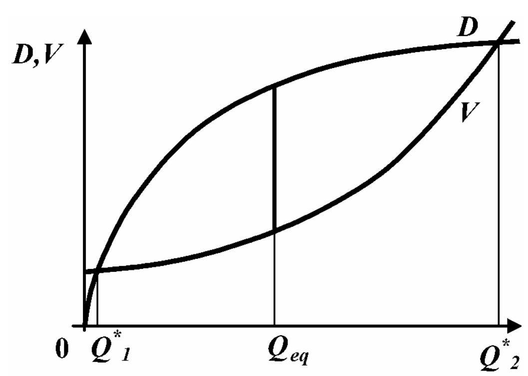

Fig. 1. Break-even points for a non-linear function cost

In this case function $D(Q)$ increases slower, as the sale of additional quantities of the production at the market requires that prices reduce gradually (Fig. 1). The function $V(Q)$ comes not from the zero (because of the fixed costs) and keeps rapidly increasing, as the expansion of production needs capital investments as well as attraction of more expensive resources, etc.

Because of these nonlinearities in Fig. 1, there are two break-even points Q^*_1 and Q^*_2 (the points of intersection of the curves D and V), in contrast to the standard linear case, when there is one such break-even point and the only task is to locate it. The maximum profit herewith lies in the area between these two points.

Let's consider a simplified (not including raw product suppliers) supply chain (Fig. 2) consisting of a manufacturer that produces volume Q and a transport company, which delivers it to the consumer market at tariff t, wherein price P is formed by the function $P = b - aQ$, where b is the highest possible price in the market, and a indicates the demand elasticity [2]. Each of the chain members maximizes its own profit.

The producer profit π represents the difference between the income PQ that he receives in the market and production costs that can be defined by means of quadratic dependence $V = v_0 + v_1 Q + v_2 Q^2$, as well as the cost of product delivery to the market tQ [3]:

$$\pi = (b - aQ)Q - v_0 - v_1 Q - v_2 Q^2 - tQ \to \max_Q . \quad (1)$$

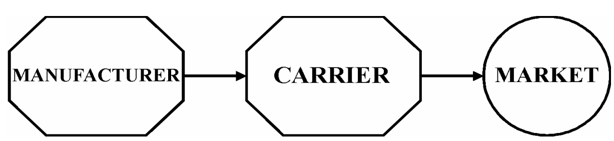

Fig. 2. Supply chain

The transportation company profit F is formed as the difference between the income from the product delivery to the market tQ and the transportation costs to be defined likewise by the quadratic dependence:

$$F = tQ - c_0 - c_1 Q - c_2 Q^2 \to \max_t . \qquad (2)$$

The fixed costs v_0 and c_0 are not to be taken into account hereinafter, as they do not affect the control parameters optimization under analysis.

In order to find the equilibrium tariff and the volume of supply it is proposed to use methods of game theory, in particular — to find the Stackelberg equilibrium balance [4]. The classical methods of finding the above mentioned balance imply the linear cost function of the participants and allow to maximize profits by focus on the private control parameters (the volume of delivery for the producer and the tariff for the transport company) [5]. The nonlinear cost function allows to optimize profits using the control parameter of another chain member. For example, the manufacturer can optimize his profits by tariff control. As the transportation company's interests are taken into account within the model, rate equal to zero will not be set.

The first case under consideration includes the tariff rate fixed by carrier, who maximizes its own profit and know the reaction of the manufacturer on its own tariff value (i.e. the carrier is the leader of the game in terms of the game theory terminology).

To find an optimal function of response of the manufacturer to the tariff t, the first derivative of (1) with respect to Q will be equated to zero and the delivery volume will be expressed as

$$Q = \frac{b - v_1 - t}{2v_2 + 2a} . \qquad (3)$$

Then the transportation company as a leader substitutes (3) in its profit function (2), which becomes a function of one variable t. To maximize the profit function, we need to equate its first derivative with respect to t to zero, wherefrom the equilibrium rate is

$$t^T = c_1 + \frac{(a + v_2 + c_2)(b - v_1 - c_1)}{2v_2 + 2a + c_2} , \qquad (4)$$

i.e. it consists of a linear component of the transport cost c_1 and a certain profit margin.

The optimum manufacturer supply volume as of a follower is defined by substitution (4) into (3):

$$Q^T = \frac{b - v_1 - c_1}{2(2v_2 + 2a + c_2)} . \qquad (5)$$

Therefore the equilibrium profits of participants are:

$$\pi^T = \frac{(b - v_1 - c_1)^2 (v_2 + a)}{4(2v_2 + 2a + c_2)^2} ; \qquad (6)$$

$$F^T = \frac{(b - v_1 - c_1)^2}{4(2v_2 + 2a + c_2)} . \qquad (7)$$

Dividing (7) by (6), we obtain $\frac{F^T}{\pi^T} = 2 + \frac{c_2}{v_2 + a}$, i.e. the transportation company profit, as of a leader, at such carriage exceeds the manufacturer's sales profit minimum twice (in case if the equilibrium sale price is set).

If the transportation tariff is set by the manufacturer as the leader, he needs to know the most optimal transportation volume for his transportation company (as the follower). The optimal response function (the desired traffic volume) of the transportation company to a fixed tariff has the following form:

$$Q = \frac{t - c_1}{2c_2} . \qquad (8)$$

Now, the leading manufacturer substitutes function (8) that is already familiar to him into the function of his profits (1):

$$\pi = \frac{(t - c_1) \cdot (2c_2 (b - v_1 - t) - (t - c_1)(v_2 + a))}{4c_2^2} \to \max_t . \qquad (9)$$

To maximize (9) we equate its first derivative with respect to t to zero and determine the equilibrium rate to be set by the leading manufacturer to the transportation company:

$$t^B = c_1 + \frac{c_2 (b - v_1 - c_1)}{a + v_2 + 2c_2} . \qquad (10)$$

The equilibrium volume of the products transportation is obtained by substituting (10) into (8):

$$Q^B = \frac{b - c_1 - v_1}{2(a + v_2 + 2c_2)} . \qquad (11)$$

As a result of comparison of (4) and (10), the transportation company is to establish a higher rate than the manufacturer:

$$t^T - t^B = (b - v_1 - c_1)\left(\frac{1}{2} - \frac{c_2 (a + v_2)}{4(2v_2 + 2a + c_2)(v_2 + a + 2c_2)} \right) > 0,$$

but the difference in rates will not exceed half of the economic potential of the supply chain $A = b - v_1 - c_1 > 0$.

When the traffic rate is to be set by the manufacturer, the equilibrium participants' profits will be

$$\pi^B = \frac{(b - v_1 - c_1)^2}{4(v_2 + a + 2c_2)^2}\,; \qquad (12)$$

$$F^B = \frac{c_2(b - v_1 - c_1)^2}{4(v_2 + a + 2c_2)^2}\,. \qquad (13)$$

The second case deals with the situation when the transportation rate is fixed not by the chain participants, but by the legislation or a multimodal operator. The code name "center" is used to refer to the third-party organization to perform the functions of the transportation rates establisher. According to [6], the equilibrium parameters of the participants in this situation will coincide with the case when the members integrate vertically in order to increase their profits. The integration gives rise to a single income function for both parties, that includes the costs of the manufacturer and the transportation companies at a time:

$$\mu = (b - aQ)Q - v_1Q - v_2Q^2 - c_1Q - c_2Q^2 \to \max_{Q}\,. \qquad (14)$$

It should be noted that, in case of profit function of the integrated members (14), there is no tariff for transportation and the transportation is carried out at the cost price of the transportation company.

The equilibrium level of product supply in case of the participants integration of the above stated chain can be similarly defined as

$$Q^O = \frac{b - v_1 - c_1}{2(v_2 + a + c_2)}\,. \qquad (15)$$

The appropriate equilibrium profit is

$$\mu^O = \frac{(b - v_1 - c_1)^2}{4(v_2 + a + c_2)}\,. \qquad (16)$$

The integration of the participants gives rise to the question of further distribution of the profit jointly received by the manufacturer and the carrier. As it is indicated in [6], the equilibrium (15–16) can be obtained not only through the integration, but under the economic independence of the participants as well, if the transportation rate t is set centrally to make ratio $\pi'(Q^O) = C'(Q^O)$, i.e. equal to the amount of the boundary income to the boundary costs.

The profit sharing is plotted in Fig. 3. The segment AB shows the value of the profits of all enterprises with the equilibrium output supply Q^O. Thus, $\pi'(Q^O)$ and $C'(Q^O)$ that are equal to the tariff, make the slope of the tangent to the curves of the manufacturer profit and the carrier cost, and the line 0D being parallel to the tangents divides the segment AB into the proportions corresponding to the distribution of the total profit between the manufacturer and the carrier.

It should be noted that the slope of 0D to the axis of Q reflects the value of the tariff to be determined by the transportation company: the larger the angle, the higher the transportation rate is.

Thus, to achieve the maximum possible volume of deliveries and total profit of economically independent (but technologically related) enterprises, it is not necessary to integrate the above stated members of the logistic chain. The equilibrium transportation rate is enough to be centrally established for them.

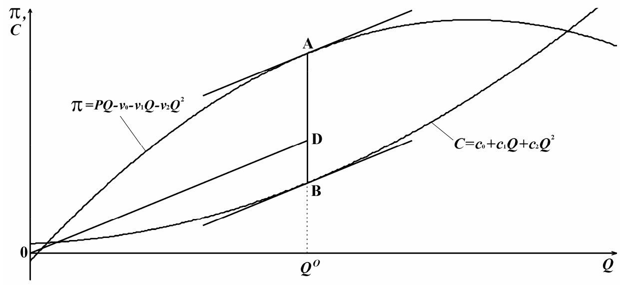

Fig. 3. Profit-sharing mechanism for the integration of participants

The three situations stated above need careful consideration (the traffic rate to be set by the carrier, the manufacturer or third-party organization – the "center") by the numerical example. Let the demand function be $P = b - aQ = 700 - 0{,}14Q$, the manufacturer function cost be $V = v_0 + v_1Q + v_2Q^2 = 6000 + 25Q + 0{,}5Q^2$, and the carrier function cost be $C = c_0 + c_1Q + c_2Q^2 = 7000 + 20Q + 0{,}35Q^2$. All cost functions and optimization are valid during the period of transportation.

Fig. 4 displays the equilibrium of the participants if the carriage tariff is established by an economically independent transportation company. Fig. 4 shows the profit function of the manufacturer without any transportation costs, to eliminate the influence of the transport tariff when comparing different situations:

$$\pi = PQ - v_0 - v_1Q - v_2Q^2 =$$
$$= (700 - 0{,}14Q)Q - 6000 - 25Q - 0{,}5Q^2.$$

It is obvious that the manufacturer's profit in this case is lower than in case of the tariff establishment made by the "center" ($A_1D_1 < AD$). The total profits of the supply chain members reduce as well ($A_1B_1 < AB$).

If the tariff is to be established by the "center", this results in the coincidence of the desires of the manufacturer and the carrier in point $Q^O = 330{,}8$ (conv. units), at some average transportation rate.

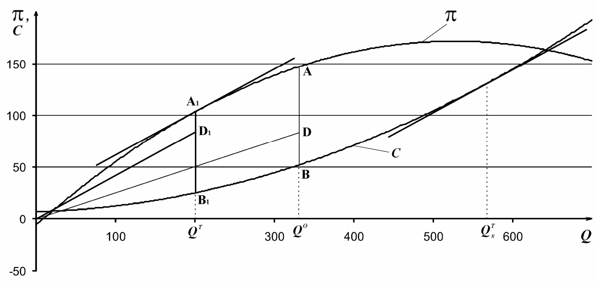

Fig. 4. Equilibrium when the tariff is established by carrier

If the right to appoint the tariff is given to the carrier, he will set a higher rate (the slope of $0D_1$ is greater than the slope of 0D). This tariff makes it unprofitable for the manufacturer to supply the former volumes of the products and the point Q^O shifts to a new equilibrium point $Q^T = 200{,}9$ (conv. units). Nevertheless, it is profitable for the transportation company to transport larger quantity of the products $Q_x^T = 568{,}3$ (conv. units) (this point can be found by constructing a tangent to the cost function of the carrier *C*, that is parallel to the tangent at A_1). But his desire is limited by the volume, determined by the manufacturer, and the ultimate equilibrium amount of cargo carried shifts to Q^T. Even when the establishment of the transport tariff is the prerogative of the carrier, the tariff does not increase indefinitely. Its optimal value is to be determined, and interests of the producer are taken into account.

In the above considered situation the earnings of the transport company exceeds the manufacturer's revenue because of an overvalued level of the tariff ($D_1B_1 > A_1D_1$ or 58.8 thousand conv. units. of carrier's profit against 19.8 thousand conv. units. of manufacturer's profit, if the cost of delivery to be taken into account). It results in the equilibrium to be established at the point Q^T, as the profit of the leading carrier exceeds the profit when the rates are set by the "center".

If a manufacturer assigns transportation tariff (Fig. 5) the opposite situation takes place.

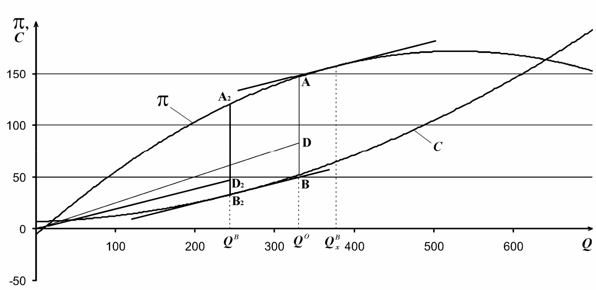

Fig. 5. Equilibrium when the tariff is established by manufacturer

He sets a rather low rate (line $0D_2$) with the purpose to carry a large quantity of products thereby ($Q_x^B = 378{,}1$ conv. units). But it is unprofitable for the transportation company to organize transportation for such a low tariff and the company agrees only to the volume of $Q^B = 244{,}4$ conv. units, where the equilibrium of the supply chain is established thereby.

In this case, the manufacturer's profit (as well as the total profit of the whole chain) appears lower than under the "center's" equilibrium. It should be noted that the manufacturer receives more profit than the carrier ($A_2D_2 > D_2B_2$).

Conclusion. Thus, the use of non-linear costs of the chain members allows to find the equilibrium parameters for different variants of tariff setting. From the standpoint of the supply chain, the balance between the interests of industrial and transportation enterprises is advisable. It can be provided by their integration or introduction of a third party to coordinate organizations' activities and set transportation rates.

However, in practice, the tariff is more frequently set directly by the transportation company, thus it is necessary to look for further ways to optimize the interests of other participants in case of the carrier's economic independence.

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