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Portfolio Creation Using Graph Characteristics and Testing Its Performance

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ABSTRACT

The aim of this paper is to propose a method for selecting underlying assets for the investment portfolio so that we can achieve a maximum expected return with minimal risk. For this purpose, we describe the portfolio creation process using the minimum spanning tree method, a graph theory tool. Using this theory, we select individual stocks from the Dow Jones Industrial Average index which we include in the portfolio and then compare the performance of such a portfolio with three alternative investments. The first alternative investment is a purchase of the entire Dow Jones Industrial Average index, the second alternative investment is to create a portfolio according to the Markowitz portfolio theory, and the third alternative investment is to create a portfolio through a random selection of index-forming stocks. The comparison was made mainly with regard to the average annual return and volatility of portfolios. In order to ensure the objectivity of the results, we divided the data into two disjoint sets. One of them was a training set, on which we built the portfolio, and the other one was a testing set used for performance testing. In the article we present that our portfolio can achieve results at least as good as those of the given alternative investments. In particular, this method also enables individual investors to determine how many underlying assets should be included in the portfolio.

INTRODUCTION

One of the most important objectives of every investor is to maximize the value of the investment with respect to risk. In this context, the appropriate choice of investments and strategy is crucial. Creating a suitable investment portfolio is a challenge for any individual investor. This is particularly true today when a wider than ever range of investment tools and strategies is available to every investor. Also in

many cases, investors face the question of how many underlying assets to choose to build their portfolios. However, there is no clear guidance for their selection.

In this paper, we describe a process of how to build a stock portfolio with the use of graph theory characteristics. Specifically, we use the minimum spanning tree theory. Using this theory, we select specific stocks from the Dow Jones Industrial Average (DJIA) index in the training period and use them to create an investment stock portfolio. Subsequently, we compare the performance of our portfolio with three chosen alternative investments in the test period. The first alternative investment is a purchase of the entire Dow Jones Industrial Average index, the second alternative investment is to create a portfolio according to the Markowitz portfolio theory, and the third alternative investment is to create a portfolio through a random selection of index-forming stocks. Through this comparison, we try to prove the pertinence and competitiveness of our strategy.

1. LITERATURE REVIEW

The foundations of the modern portfolio creation process were laid by Harry Markowitz. The theory is known as “Modern portfolio theory” or Markowitz's mean-variance model. Markowitz pointed out the possibility of reducing overall portfolio risk through diversification. Portfolio risk, measured by its variance, is not only dependent on the variance in the returns of the individual assets in the portfolio, but the contribution of the individual assets to the risk of the entire portfolio and their correlation is important. (Markowitz, 1952, 1959)

Modern portfolio theory has been used to this day. This theory has a lot of extensions and developments. One of the most significant developments of Markowitz's theory was brought forward by William Forsyth Sharpe. He defined the so-called Capital Asset Pricing Model (CAPM) in his work. (Sharpe, 1964) Markowitz and Sharpe were awarded the Nobel Prize in 1990 for their pioneering works. More recently, Hany Fahmy extended the classical mean-variance model to a time dimension that allows ex-post trading. He demonstrates that the proposed model can explain many of the observed time-related anomalies of stock returns and shows that long-term trading strategies are more profitable for rational investors under perfect information. (Fahmy, 2020) In our paper, we use the theoretical background from Markowitz's and Sharpe's works to create an alternative investment strategy.

It should be noted that there are also many works dealing with shortcomings of modern portfolio theory. Some of them argue, that mean-variance optimization is not as efficient in comparison with naive diversification, which equally invests across N assets. (DeMiguel et al., 2009) Zhifeng Dai and Fei Wang (2019) introduce a sparse mean-variance portfolio model as a better optimization portfolio model, intending to reduce the impact of parameter uncertainty and estimation errors of the mean-variance portfolio model. Dayong Zhang et al. (2018) reviews several modifications that improve the performance of the mean-variance model, including dynamic portfolio optimization, portfolio optimization with practical factors, robust portfolio optimization, and fuzzy portfolio optimization. Raymond Kan and Guofu Zhou (2007) show the advantages of the Bayesian approach under a diffuse prior, which outperforms the standard mean-variance framework. There are a lot of other authors dealing with the Bayesian approach. For example, Bauder et al. (2020) examines the optimal portfolio choice problem when the asset returns distribution parameters are unknown, using in their approach the Bayesian posterior predictive distribution.

Over time, new approaches to portfolio creation and optimization began to emerge. In our work, we use the minimum spanning tree theory to build our portfolio. One of the first authors introducing this theory was Rosario Mantegna. In his pioneering work, he studied the hierarchical structure of the US stock market (Mantegna, 1999). Graph-based methods have become a useful tool used in the investment portfolio creation process and there are many authors dealing with this topic. For example, Lee and Nobi (2018) study the network structures of global stock markets around the 2008 global financial crisis. They generate complex networks such as the threshold network and the minimal spanning tree. Mansooreh Kazemilari et al. (2019) examines the factors that have been driving renewable energy stock prices in the US markets, using the minimal spanning tree and sub-dominant ultrametric methods. Erick Limas (2019) uses minimal spanning trees and hierarchical trees to study the exchange rates in Latin America.

He analyses nine Latin American currencies and tries to identify clusters of exchange rates with similar co-movements. Lee and Woo (2019) analyze the dynamics of the stock network to recommend the stock portfolio. For forecasting the dynamics, they use an innovative measure - the cohesion of the stock market network induced by the correlation of stocks. They show that cohesion affects the change of the stock prices and can be used for stock portfolio recommendations. The new possibility of portfolio creation using the minimum spanning tree method is described by Danko. This new approach, based on graph theory, is suitable for an individual investor to create an investment portfolio (Danko et al., 2020). In one of the latest research authors study financial markets during the global pandemic of COVID-19, investigating systemic connections among specific countries using graph theory and minimum spanning tree method before the pandemic announcement and after (Zhang et al., 2020).

There are also other approaches used in the portfolio selection process. One of them is using neural networks. Fernández and Gómez (2007) apply artificial neural network models in their work to obtain the efficient frontier, which is connected to the portfolio optimization problem. More recently, Pang and others introduce an innovative neural network approach for stock market prediction. They claim that traditional neural network algorithms may incorrectly predict the stock market. Instead, they propose the deep long short-term memory neural network with an embedded layer and the long short-term memory neural network with an automatic encoder for stock market predictions (Pang et al., 2020). Similarly, Zhou proposes an improved neural network model to predict stock market trends, introducing a new hybrid end-to-end approach containing two stages – the empirical mode decomposition and factorization machine-based neural network. For the demonstration of the accuracy of the new approach, real datasets were used. (Zhou et al., 2019)

Recently, the passive form of investing has been increasingly coming to the forefront among investors. It consists of a purchase of an entire index. The benefits of passive investing are highlighted by Burton Malkiel. He claims that the records support passive investment strategies in various markets. (Malkiel, 2003) Edwin Elton and others study the factors important in explaining differences across funds and ETFs following the same index. They demonstrate how to select a passive investment that has a high probability of having the best performance in the following years (Elton et al., 2020). We use the passive form of investing in our paper as an alternative strategy.

2. DATA AND METHODOLOGY

We analyzed daily closing prices for the shares forming the Dow Jones Industrial Average as of 2 July 2019. As of that date, the index consisted of 30 companies listed in Table 1.

Table 1. Tickers and names of analyzed companies forming the DJIA index at the date of analysis

MMM	3M	XOM	Exxon Mobil	NKE	Nike
AXP	American Express	GS	Goldman Sachs	PFE	Pfizer
AAPL	Apple	HD	Home Depot	PG	Procter & Gamble
BA	Boeing	IBM	IBM	TRV	Travelers Companies Inc
CAT	Caterpillar	INTC	INTC Intel	UTX	United Technologies
CVX	Chevron	JNJ	Johnson & Johnson	UNH	UnitedHealth
CSCO	Cisco	JPM	JPMorgan Chase	VZ	Verizon
KO	Coca-Cola	MCD	McDonald's	V	Visa
DIS	Disney	MRK	Merck	WMT	Wal-Mart
DOW	Dow Chemical	MSFT	Microsoft	WBA	Walgreen

Source: own research

The aim is to create a portfolio for an individual investor, made up of a one-time purchase at the beginning, while we are not considering short selling. The methodology described below could also be applied in automated trading systems, which are currently very widespread.

We conducted the analysis in the period from 2 January 2009 to 2 July 2019. We chose this time period because of the availability of data for all the shares listed in the market index. This is the period long enough to cover all potential moods and market developments. We have to say that the index composition is dynamically changed over time and there is about one revision of this composition on average per year, but it does not affect our analysis.

From the daily closing prices of the stocks forming the analyzed index, we calculated the daily returns in the standard way:

$$r_i = \frac{P_i - P_{i-1}}{P_{i-1}} [\times 100\%] \quad (1)$$

where p_i is the closing price at the time i , $p_{(i-1)}$ is the closing price on the previous trading day (at the time $i - 1$) and r_i represents the daily rate of return at the time i . Thus, every single stock from the analyzed index is represented by a daily return vector. In the same way, we calculated the daily returns of the index itself based on DJIA values.

The aim is to propose a portfolio composition based on the analysis of profitability for a particular year and then to evaluate its performance based on the knowledge of price development of individual components of the portfolio and the index itself in the following year. The calculation logic is illustrated by the following timeline (Figure 1):

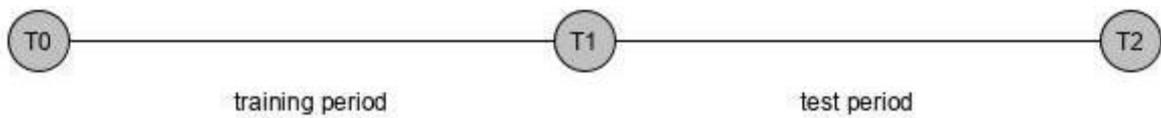


Figure 1. Timeline

Source: own research

Based on the analysis of daily returns in the period from T0 to T1, a portfolio creation proposal was prepared - identification of the individual stocks from the DJIA index to create a portfolio. Subsequently, this portfolio performance was tested in the period from T1 to T2. For the same period, its performance was compared with the performance of the market index, with an alternative investment in the form of using Markowitz's portfolio theory and with a portfolio based on a random selection of stocks. Therefore, this approach is also very suitable for an individual investor as it is a practical example of how an investor can make a portfolio decision at time T1 for period T1 - T2, only based on historical returns from period T0 - T1.

In our case, we have chosen an equidistant time interval of one business year, which is approximately 252 business days. We started by considering the period of the last 252 trading days until 2 July 2019 as the last test period (T1 - T2) and the period of 252 trading days immediately preceding it as the training period (T0 - T1), based on which we build the portfolio. Afterward, we continued to move one day back and formed pairs of such intervals in the same way until we were in a situation where the start of the training period was equal to 2 January 2009, which is the starting day of our analysis. In this way, we obtained 2017 pairs of testing and training intervals for the analyzed period. For better understanding, we illustrate the algorithm on the timeline (Figure 2), which clearly shows that the testing and training periods never overlap for a particular pair and that consecutive pairs overlap in (252-1) values for both the testing and training periods. For better understanding, we have to mention, that the training period represents the computational part of our analysis and the testing period represents the experimental part of our analysis.

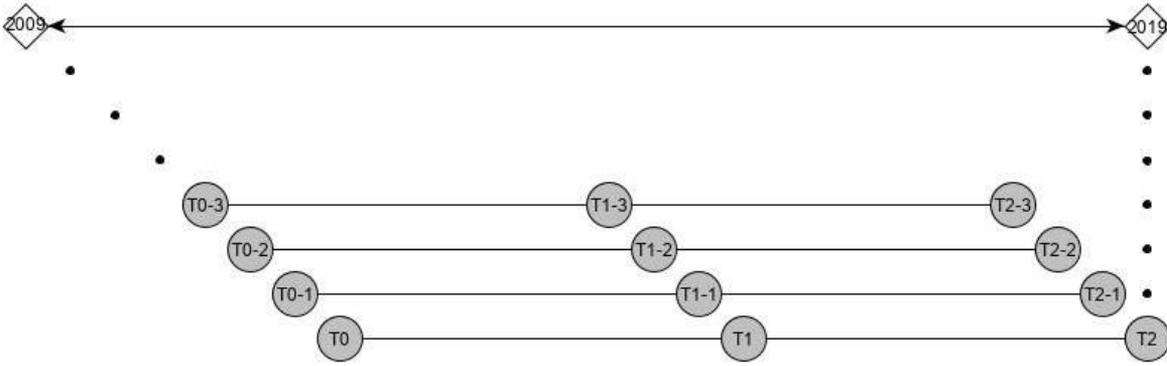


Figure 2. Calculation logic

Source: own research

For each training period, we calculate the correlation between the stock pairs using the Pearson correlation coefficient given in Equation 2.

$$\rho_{i,j} = \frac{cov_{i,j}}{\sigma_i \cdot \sigma_j} \quad (2)$$

where $\rho_{i,j}$ is a correlation coefficient between returns of stocks i and j , $cov_{i,j}$ is the covariance between these returns and σ_i is the standard deviation of the stock's i return.

In this way, we obtain 2017 correlation matrices (for each training period). These are symmetrical square matrices of dimension $n \times n$, where n represents the number of stocks that make up the analyzed index (in our case 30), and the value in the i -th row and j -th column represents the Pearson correlation coefficient between i -th and j -th stock. The correlation matrices were converted to *distance matrices* using the *ultra-metric*:

$$d(i,j) = \sqrt{2 \times (1 - \rho_{i,j})} \quad (3)$$

where $d(i,j)$ is the distance. The reason why we have converted correlation matrices to distance matrices is that the correlation coefficient of a pair of stocks cannot be used as a distance between the two stocks (correlation coefficient reaches a negative value, the correlation of two identical vectors is not zero and so-called triangle inequality – subadditivity is not valid). It does not fulfill the three axioms that define a Euclidean metric (the identity of indiscernibles, symmetry, and subadditivity). For further details on the ultra-metric see Mantegna (Mantegna, 1999).

Accordingly, we obtain 2017 distance matrices (for each training period). These are symmetric square matrices of dimension $n \times n$, where n represents the number of shares forming the analyzed index (in our case 30), the value in the i -th row and j -th column is the distance between the i -th and j -th stock given by Equation 3. Distance matrices formed the basis for the application of a discrete mathematical tool – graph theory. Specifically, we constructed complete graphs that have been defined as follows: the set of vertices are the specific stocks forming the Dow Jones Industrial Average and the set of edges is given by the distances from the distance matrix.

From complete graphs, we have constructed the so-called minimum spanning trees of this graph. The minimum spanning tree is a subgraph of a complete graph that is continuous, does not contain cycles, and has one edge less than the number of vertices. For a subgraph that meets these characteristics, there is a unique path between each of its two vertices. In the case of a complete graph, there is n^{n-2} spanning trees, where n represents the number of vertices - in our case 30 (Cayley, 1854). Among these spanning trees, we can find the only one that has the lowest edge evaluation. The statistical programming language R, specifically the *igraph* package in which the analysis was performed, uses the Prim algorithm to calculate the minimum spanning tree of the graph (Jarník, 1930; Prim, 1957).

After estimating the minimum spanning tree for each training period, we have identified those vertices that have a degree of one. The degree of a vertex is the number of edges that are incident to the

vertex. We assume that these stocks have a higher diversification potential and by combining them we can get a significantly diversified portfolio. This is based on the knowledge of Onnela and others. They found that there is a link between the vertices forming the low-risk portfolios and their corresponding vertex attributes in the minimum spanning tree of the graph. The authors argue that in any of the minimum spanning trees, the vertices representing the shares with the greatest diversification potential are located at its periphery. (Onnela et al., 2003) For this reason, we create a portfolio from stocks that represent vertices with degree one, which we consider to be at the periphery.

After identifying vertices with degree one for each training period, we build a portfolio with equal weights of $1/k$, where k is the number of stocks with degree one. In this way, based on the correlation of returns in the period $T_0 - T_1$, we build a portfolio with equal weights consisting of k stocks at time T_1 . Afterward, for the period $T_1 - T_2$, we quantify the performance of our portfolio on new data and compare it with alternative investments. We want to know how the value of our portfolio will change in the testing period (at time T_2 compared to time T_1).

The first alternative investment is the purchase of the entire Dow Jones Industrial Average index at time T_1 and its holding until time T_2 . The second alternative investment is based on Markowitz's portfolio theory. The idea is that in the period $T_0 - T_1$, based on the historical prices and returns, the weights for the so-called optimal portfolio are calculated. It is a portfolio that would provide the investor with the best return on the given risk during the period $T_0 - T_1$. The optimization parameters can be adjusted to the investor's requirements. Using Markowitz's optimal portfolio theory, we are looking for a distribution of the weights of individual stocks from the DJIA index (w_1-w_{30}) in time T_1 . In our case, we are looking for weights in the portfolio that would ensure the best ratio of expected return to risk, characterized by the standard deviation of these returns. The weights should maximize the Sharpe ratio. Sharpe ratio is calculated as the return of the portfolio, divided by the standard deviation of the portfolio's excess return.

$$S_p = \frac{E(r_p)}{\sigma_p} \quad (4)$$

The return of the portfolio is calculated by the following formula,

$$R_p = \sum_{i=1}^N w_i r_i \quad (5)$$

where R_p represents the expected return of the portfolio, w_i the weight of the i -th asset in the portfolio, r_i represents the expected return of the i -th asset.

Standard deviation is calculated by the following formula,

$$\sigma = \sqrt{w \cdot \Sigma \cdot w^T} \quad (6)$$

where w represents row weight vector, Σ the covariance matrix of profitability and w^T transposed row weight vector (column weight vector).

The optimization problem is

$$\frac{\sum_{i=1}^N w_i r_i}{\sqrt{w \cdot \Sigma \cdot w^T}} \rightarrow \text{maximum} \quad (7)$$

with optimization conditions:

- we exclude the risk-free rate,
- the weights cannot be negative, so there is no possibility to have „short“ positions in the portfolio
- the portfolio can contain from 1 to 30 shares, weights could also be zero,
- the sum of weights is equal to one.

For solving the optimization problem, the R package *quadprog* was used. The weights of shares computed by this method represent an alternative investment at time T_1 . Again, our task is to find out how the value of the portfolio so constructed will change in the test period (at time T_2 compared to time T_1). The third alternative investment is a situation where we would randomly select some shares at time T_1 and build a portfolio with the same weight.

3. RESEARCH RESULTS

In the following section, we will present the basic results we have obtained by comparing the performance of our portfolio and alternative investments. We calculated the performance of all investments to average annual returns so we could compare them against each other. The strategies proposed by us achieved in the test periods an average annual yield of 11.11% with a standard deviation of 0.0863, and the coefficient of variation at 77.65%. The most important results of the comparison are summarized in Table 2.

Table 2. Comparison of the proposed portfolio with alternative investments

Strategy	Average annual yield	Standard deviation	Coefficient of variation
Minimum spanning tree	11.11%	0.0863	77.65%
DJIA index	10.79%	0.0852	78.95%
Markowitz portfolio theory	11.32%	0.0862	76.16%
Random stocks selection	11.31%	0.0933	82.49%

Source: own research

3.1 Comparison with DJIA index

If we look at the market yield that we would get by purchasing the entire Dow Jones Industrial Average index, its average annual return is 10.79% with a standard deviation of 0.0852, which represents a coefficient of variation of 78.95%. The sample Pearson correlation coefficient for these two investments is 0.8882. Our strategy was better than the market index in 1058 of the 2017 time periods - a 52.45% success rate. The simultaneous development of the return of both these investments is shown in Figure 3. Using our proposed method, we have shown we can achieve a better return than the market index in terms of expected annual profitability. Our portfolios are also better in terms of relative variability (smaller coefficient of variation).

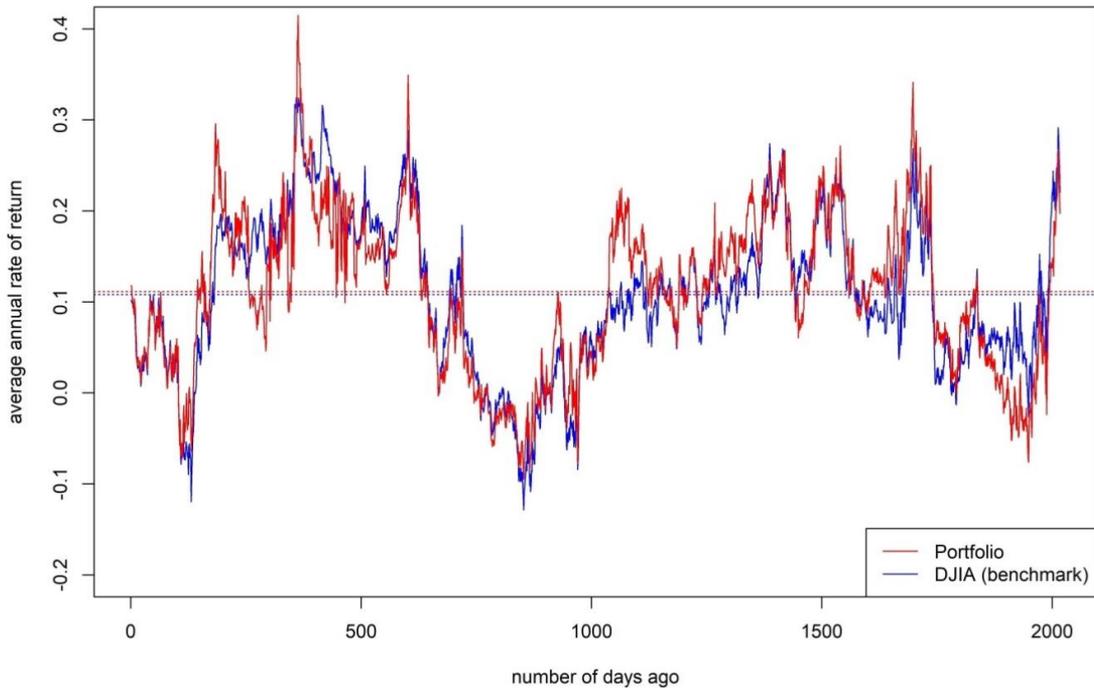


Figure 3. Annual average return of the portfolio compared to DJIA

Source: own research

3.2 Comparison with Markowitz optimal portfolio theory:

Using historical data from the training period we applied Markowitz's approach at time T1 and looked for the portfolio with the highest value of the Sharpe ratio (expected return to risk ratio - the inverse value of the coefficient of variation). Based on the weights obtained this way, we built the portfolio at time T1 and calculated its return over the period T1 - T2. We found that the average annual return on such an investment is 11.32% with a standard deviation of 0.0862. The coefficient of variation stands at the level of 76.16%.

The sample Pearson correlation coefficient for these two investments is 0.9159, indicating an even closer positive linear relationship between this pair of investments. Our strategy was better than Markowitz's optimal portfolio strategy in 918 of 2017 time periods, with a 45.53% success rate.

The simultaneous development of returns of both investments is shown in Figure 4. Using our proposed method we have shown we cannot achieve a better return, in terms of the annual expected return, than Markowitz's portfolio. However, there are no big differences, and the computational and technical complexity of Markowitz's approach is more challenging than our method, which presents an advantage for our approach.

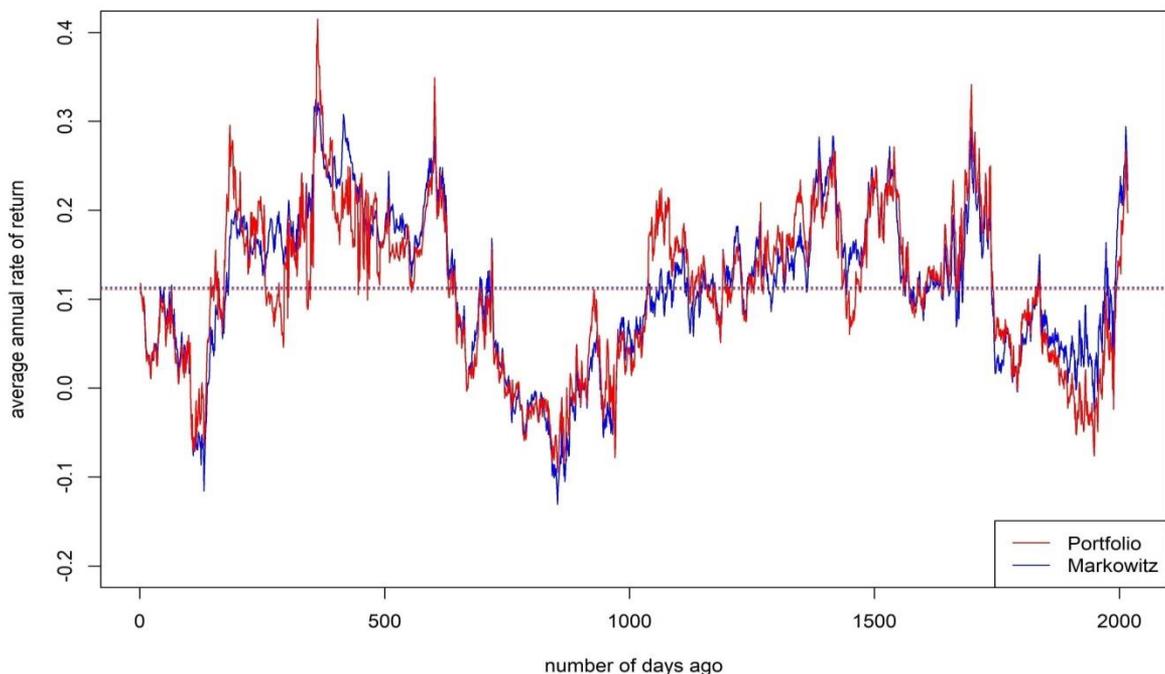


Figure 4. Annual average return of the portfolio compared to modern portfolio theory (Markowitz)

Source: own research

3.3 Comparison with a portfolio constructed through a random selection of stocks

Since the portfolio creation method we proposed is based on selecting stocks using the minimum spanning tree consequently giving all identified shares the same weight, we decided to create an alternative portfolio in the same way. However, we will not use an objective criterion based on the placement at the periphery of the minimum spanning tree, but a random selection of shares. For example, if we used our method and selected 16 stocks with a degree of one in a training period and assign a weight of 1/16 to each of that stocks, then in the same period we will randomly select 16 stocks and assign also the weight of 1/16 to each of them. It means that the probability of being chosen for each stock is m/n , where m is a number of stocks selected using our method (in the example above 16) and n is the total number of stocks (in our case 30). We chose such a strategy to compare portfolios consisting of the

same number of shares and to create them in almost the same way (once according to objective criteria and once completely randomly) to show that our approach to portfolio creation brings better results.

The average annual rate of return for such an investment is 11.31% with a standard deviation of 0.0933. The coefficient of variation is 82.49%.

The sample Pearson correlation coefficient for these two investments is 0.8462. Our strategy was more successful than random selection strategies in 991 of the 2017 time periods, a 49.13% success rate.

The simultaneous development of returns of both of these investments is shown in Figure 5. Based on the graphical visualization and calculations of the basic portfolio characteristics, we can see that despite the relatively similar expected return on both investments, the random investment is more volatile (in terms of absolute variability given by the standard deviation and the relative variability given by the coefficient of variation). For this reason, we consider the portfolio strategy we proposed to be better than the strategy of selecting stocks randomly.

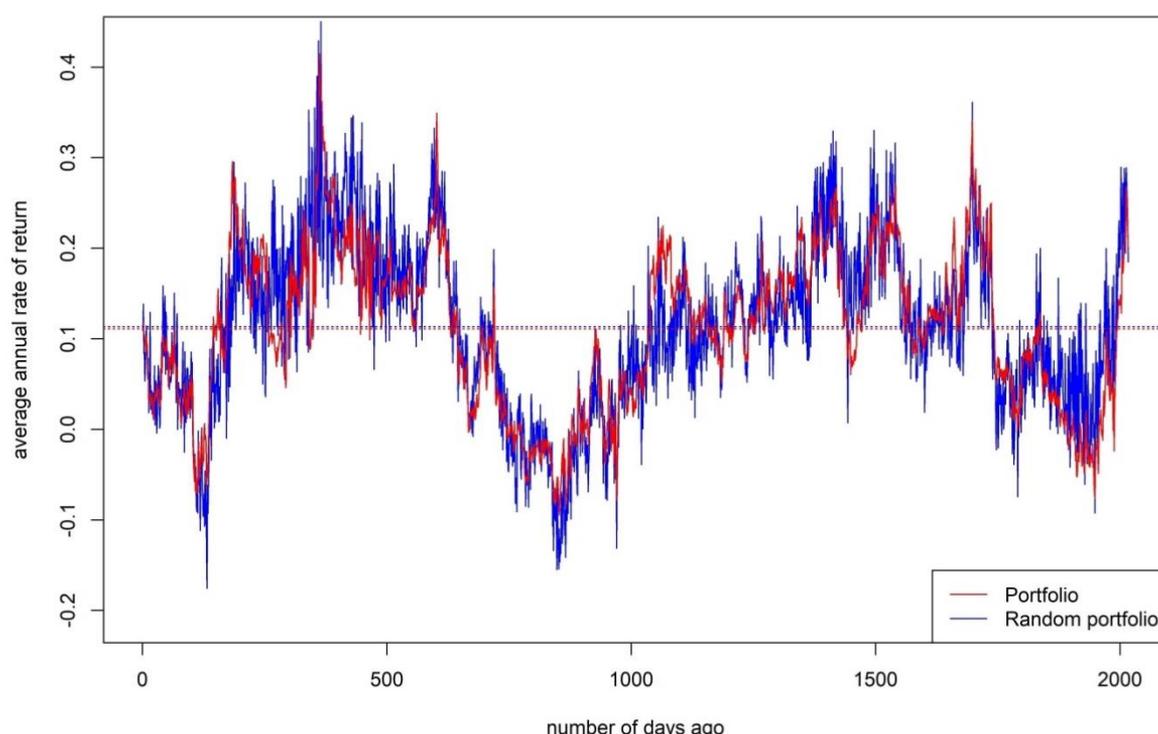


Figure 5. Annual average return of the portfolio compared to the random portfolio

Source: own research

CONCLUSIONS

In this work, using the minimum spanning tree theory, we proposed an investment stock portfolio suitable for an individual investor. In applying our approach, we selected individual stocks from the Dow Jones Industrial Average index in the training period and created an investment portfolio consisting of them. In the test period, we compared the success of the proposed portfolio with the three alternative investment strategies. The comparison was made mainly concerning the average annual return and volatility of portfolios. The results of the comparison are summarized in Table 2.

The proposed portfolio achieved a better average annual return and coefficient of variation than the entire Dow Jones Industrial Average index. Our portfolio did not outperform the portfolio created according to Markowitz's approach. That achieved a higher average annual return and a better coefficient of

variation. However, the differences were altogether inconsiderable and therefore we do not consider them to be very significant for an investor. Furthermore, the simpler technical and computational complexity of our approach makes it more accessible for investors. The portfolio made through the selection of a random stock achieved a better annual average return, but at the same time had a worse coefficient of variation. Again, the difference in returns is very small, and a better coefficient of variation supports the strategy we proposed.

In practice, our proposed method using minimum spanning tree, and specifically the vertices that have a degree of one, has real-world investing applicability and relevance. We have shown that such an approach can be competitive with other, often better-known, strategies. While in this paper we applied this approach for specific stock selecting from Dow Jones Industrial Average index, this approach can be applied more generally. The merit of this paper is evidenced in our analysis that provides guidance for portfolio creation to an individual investor, whose objective is to diversify risk and to achieve at least a market return with an active investment approach.

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